MULTICRITERIA METHOD FOR CHOOSING THE BEST ALTERNATIVE FOR INVESTMENTS

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ABSTRACT. There is evidence of mistakes in reconstruction of buildings in historical places of cities. Due to high cost of such mistakes, the choice of the best project from the several variants should be well founded. This paper presents multicriteria method for choosing the best object, called SNOD (Scale of Normalized and Ordered Differences). There are two stages in the method: preliminary formal analysis of given alternatives, performed entirely by a computer, and Decision Maker preferences elicitation. The method is based on a Verbal Decision Analysis approach. The possibility of criteria interdependence is taken into account. To evaluate the method effectiveness a statistical modeling was carried out. The method has been applied for practical cases of buildings reconstruction and supermarket locations in Vilnius. The paper is partially supported by RFBR #01-01-00514, #02-01-01077 and Program of Presidium of RAS MMIS.

KEYWORDS: decision-making problems, multicriteria method, Scale of Normalized and Ordered Differences, Verbal Decision Analysis.

1. INTRODUCTION

Problems of alternatives ranking and choosing the best alternative are close enough to each other in the sense that subsequent choosing of the best alternative of the given set allows accomplishing their ranking. However, the problem of choosing the best alternative is in general simpler and deserves special attention.

The problem of choosing the best alternative of a given set of alternatives estimated by multiple criteria is one of the decision-making problems widespread in practice. Such problems occur both in organizations and in personal life. The choice of a reform plan, corporation reorganization, a building reconstruction plan are examples of business problems of such type. Examples of personal problems are the choice of profession, an expensive purchase, etc.

In many cases choosing the best alternative one uses pair-wise comparison of alternatives and exclusion of dominated alternatives in pairs. First of all, let's note that this approach is most typical for people making decisions in personal life without use of computers. O. Svenson and H. Montgomery proposed psychological theory of the search for dominance structure [7]. According to this theory Decision Maker (DM) comparing pairs of alternatives wants to find the best one, which would excel any of others. In compliance with the theory of the search of dominance structure DM overlooks all given alternatives and chooses the one that could (by the first impression) be dominant. Then DM compares it with all the others pair-wise. If the chosen alternative wins over all comparisons then dominance structure is built. If after one of the comparisons another alternative turns out to be better then it is this alternative which is considered the potentially dominant, and all other comparisons are carried out with it.

It was also noted that the problem of choosing the best of multicriteria alternatives was complex enough for a man. For example, isolating the subset of the best alternatives the subjects could remove dominant alternatives and leave dominated ones [3].

Pair-wise comparison of alternatives attracted researchers as a normative method of decision making. It was B. Franklin who first proposed it back in 18th century in his letter to a friend [4].

The wide known disadvantage of pair-wise comparison is the possibility of cycles [10] on the set of compared alternatives.

In the network of Verbal Decision Analysis (VDA) approach there was earlier proposed method PARK [5] directed to choosing the best alternative of a group of given multicriteria alternatives by pair-wise comparisons, which allowed to solve important practical problems [6].

However, method PARK has significant disadvantages:

- It is designed for choice tasks with 3-5 alternatives, because it makes use of pair-wise comparisons of all variants.
- Criteria scales have only verbal estimations;
- The assumptions of possible operations for elicitation of DM information are not proven by the results of psychological research.

The new method for choosing the best of a group of given alternatives presented below, method SNOD, has the following features:

Firstly, it actively uses possibilities of a computer, which by definite rules without help of DM performs pair-wise comparisons of all alternatives analyzing their similarities and differences. At the same time computer prepares the most effective process of DM questioning, which creates opportunities to analyze larger groups of alternatives.

Secondly, some of quantitative criteria (for example, cost) can also be considered as native and comfortable language to express DM preferences. Therefore, the method allows to work with quantitative as well as qualitative estimations of alternatives.

Thus, the new method extends and amplifies possibilities of the other Verbal Decision Analysis methods.

2. PRACTICAL EXAMPLE

The countries of East Europe carrying out reforms urgently need methods for choosing the best object for investments in the processes of buying a building for reconstruction and subsequent sale, arrangement of supermarkets and agricultural production centres, etc [8, 9, 11].

Let's give a more detailed example of such problem. For the sake of clearness the example consists of only four alternatives, although the method SNOD can be applied to larger problems.

A company is looking for a location to construct a supermarket. The preliminary analysis has shown that there are four potential locations (Var1 – Var4). Solving the problem of choice the company management has decided

Table 1. Alternatives representing possible locations for a supermarket

	Var1	Var2	Var3	Var4
Parking capacity, max	400	300	250	150
Presence of competitors, min	1 Low	5 High	3 Medium	5 High
Population density within 1 km radius, max	200	4500	6000	7000
Price of the site, thous. Lt/1 a., min	6	16	12	20
Public transport flow, max	1 Low	3 Medium	5 High	7 Very High
Visibility from a principal street, max	5 High	5 High	3 Medium	1 Low
Communications infrastructure, max	3 Medium	3 Medium	5 High	7 Very High

to use the following criteria: the price of the site, density of population within 1 km radius, presence of competitors, communications infrastructure, car parking capacity, accessibility of the site by public transport and visibility of the store from the principal street.

The invited experts estimated the selected alternatives using either estimation scales in natural units or verbal estimations presented by marks.

Table 1 presents alternatives with estimations. The table specifies the desired direction of changes in the estimation of each of the criteria (max or min).

3. FORMAL PROBLEM STATEMENT

Given:

1. C_i – set of criteria, i = 1,., q,.. N .
2. s_q – number of estimations on the scale of the q^{th} criterion.
3. $X_q = x_i^q$ – set of estimations on the scale of the q^{th} criterion; $|X_q| = s_q$; estimations are ordered from better (first) to worse (last);
4. $Y=X_1*X_2*\ldots X_N$ – set of vectors $y_i \in Y$ in the form: $y_i = (x_1^i, x_2^i, \ldots x_N^i)$, where $x_i^q \in X_q$ and $P=|Y|$.
5. $V(y_i)$ – overall value of an alternative to DM. It is supposed that it has the following characteristics: à) there is maximal and minimal value on the Y set; á) providing that criteria are independent $V(y_i)$ grows with the improvement of estimation on any criteria.
6. $A=\{a_i\} \in Y; \quad i=1,2,\ldots,n$ – set of n 1. vectors, describing real alternatives.

It is required to choose the best alternative from A set, corresponding to the maximum value of a priori unknown function $V(y_i)$ on the basis of Decision Maker's preferences.

4. ASSUMPTIONS

The following assumptions about DM capabilities are put at the heart of the method:

1. DM is capable of comparing two multicriteria alternatives, which differ by estimations on two criteria only;
2. DM is capable of comparing two multicriteria alternatives, which differ by estimations on more that two criteria, if one of the alternatives is more preferable than the other by one criterion and less preferable by not more than three criteria.

Having compared two alternatives (A and B) DM can give one of three possible answers:

1. Alternative A is more preferable than B.
2. Alternative B is more preferable than A.
3. Alternatives A and B are equally preferable.

These assumptions about DM capabilities have the following proofs:

First assumption was repeatedly verified during application of methods of ZAPROS family [5]. It was found that people are consistent enough to accomplish this task. There were observed 2-6 contradictions per 50-60 comparisons.

The second assumption was verified in [1]. It is assumed that in the case of criteria independence the comparison of a preferable estimation of an alternative with two or three preferable estimations of another one doesn't create excessive load on short-term memory. The hypothesis was proven that DM can consistently compare alternatives which differ by three criteria.

Based on these assumptions the procedure of comparison of two multicriteria alternatives is proposed. It utilizes pair-compensation principle, when an attempt is made to counterbalance the advantages of one alternative by advantages of another one.

5. FORMAL ANALYSIS

The main goal of formal analysis of alternatives set consists in revelation of Potentially Best Alternative, which acts as a comparison standard for DM during pair-wise comparison to other alternatives. Of course it is possible to present DM the list of alternatives and ask him to point out the presumably best one. However, it is known, that if there are many enough alternatives and criteria then this task is rather difficult for a human being. Mistakes in the solution of this task can lead to extra additional comparisons. That's why it is better to choose potentially best alternative in a formal way.

Of course, in formal analysis stage we know nothing about actual DM preferences or about the importance to him of these or those estimations by criteria. However, even under these conditions formal analysis can focus DM attention on the alternative, which is somewhat better than the others.

A computer-aided analysis of the problem is carried out without DM's participation. During the analysis an assumption is made that the criteria are equally important.

Formal analysis can be divided in two stages:

The first stage consists in pair-wise comparisons of all alternatives, done by a computer using the following algorithm:

1) Normalization of estimations in each pair of compared alternatives:
   a) For quantitative scales: the mean value is found for the estimations of two compared alternatives;
   b) For qualitative scales: quality estimations are converted to marks (the order index of quality estimation is taken as its mark) and used to find "mean" (artificial) value;
   c) If the greatest value (max) is to be reached by the criterion, the estimation of the alternative is divided by the mean value; if the least value (min) is to be reached, the mean value is divided by the estimation.
2) For each of the two alternatives the sum of normalized estimations is counted.
3) The winner is the alternative having the greatest sum.

Let us make an important comment: the presented algorithm could be applied also for the group of given alternatives. But in a general case the winner for the group could be different from the winner in the pair comparisons.

The second stage consists in preparation of order of questions to DM to elicit his preferences.

This stage is also carried out entirely by the computer without DM participation.

a) Basing on de Condorce principle [2] the alternative, which has the largest number of wins in the course of pair-wise comparisons in the 1st stage, is selected. It is announced Potentially Best Alternative (PBA).
b) The alternatives, which are not Pareto-optimal, are excluded from consideration.
c) All other alternatives are ordered by formal difference from PBA, to provide gradual ascending of questions difficulty.

Thus, the goal of formal analysis is preparation of series of questions to be posed to DM that could provide:

- a minimum load for DM: the least number of questions to be posed;
- gradual ascending of questions difficulty;
- the greatest possible use of DM information.

6. PROBLEMS THAT COULD ARISE IN THE FORMAL ANALYSIS STAGE

Determining potentially best alternative in the formal analysis stage by pair-wise comparisons we utilize de Condorce principle [2]:

"The alternative, which wins over all other alternatives in the course of pair-wise comparisons, should be considered the best one".

Although from the common sense point of view de Condorce principle looks rather unquestionable, in certain cases it leads to cyclic relations on the set of alternatives. For example, let us take the following three alternatives A, B and C (the criteria are equally important, max means a direction of the preferable variation of the criterion):

	A	B	C
Criterion 1, max	1	2	3
Criterion 2, max	2	3	1
Criterion 3, max	3	1	2

It turns out that B > A, A > C, but B < C as a result of the formal analysis.

It is possible to find the condition under which cycles never occur.

Let's denote as $\sigma_1^{jk}, \sigma_2^{jk}, \cdots, \sigma_N^{jk}$ - mean values for the criteria calculated in the course of pair-wise comparison of alternatives A_j and A_k. Let's denote as $\gamma_1, \cdots, \gamma_N$ - mean values calculated for the group of given alternatives according to the above-mentioned rules. In accordance with the above-mentioned procedure, alternatives A_j and A_k are not in the relationship of Pareto-dominance, i.e. there are criteria, by which A_j exceeds A_k and vice versa. Let's denote as C_{jk}^{+} - a set of criteria, by which A_j exceeds A_k and as C_{jk}^{-} - by which it loses to it. Let A_j is selected as PBA, and it exceeds A_k by formal analysis.

Then

$$\sum_{i \in C_{jk}^{+}} \frac{x_i^j - x_i^k}{\sigma_i^{jk}} > \sum_{i \in C_{jk}^{-}} \frac{x_i^j - x_i^k}{\sigma_i^{jk}} \quad (1)$$

Statement 1: A sufficient condition of cycles absence on the set of alternatives A is satisfaction of the following conditions for any pair of alternatives A_j and A_k:

$$\gamma_i \le \sigma_i^{jk}, \quad \forall\, i \in C_{jk}^{+} \quad (2)$$

$$\gamma_i \ge \sigma_i^{jk}, \quad \forall\, i \in C_{jk}^{-} \quad (3)$$

Proof: Indeed, when (2), (3) are true the condition (1) is also true and will be the same both for pair-wise comparisons and for group comparison of all the alternatives, where cycles can never occur. Thus, cycles will be absent also for pair-wise comparisons, which was to be proved.

It is evident that statement 1 is sufficient, but not necessary condition of cycles absence.

Since in general there can be cycles one need an algorithm to determine PBA in their presence. It seems reasonable to use the following algorithm. If alternatives A_i, A_j and A_k exceed all other alternatives during a pair-wise comparison and form a cycle, then the best alternative for the group comparison of A_i, A_j and A_k should be taken as PBA. If they all are formally equal then any of them can be selected as PBA.

7. EXAMPLE: THE FORMAL ANALYSIS

Let's take our example. Table 2 presents the stage of alternatives Var1 and Var2 comparison by the formal analysis rules. The table contains the initial alternatives, their representations in the normalized form in accordance with the rules of formal analysis, the differences between the normalized estimations (the "Difference" line) and the result - sum of the components of the "difference". If the result is higher than 0 then the first alternative from the pair is better than the second one, otherwise – vice versa.

In this case Var1 is the best alternative in

Table 2. An example of formal analysis

Criteria	1 (max)	2 (min)	3 (max)	4 (min)	5 (max)	6 (max)	7 (max)
Comparison of Var1 and Var2							
Var1	400.00	1.00	200.00	6.00	1.00	5.00	3.00
Var2	300.00	5.00	4500.00	16.00	3.00	5.00	3.00
Normalized Var1	1.14	3.00	0.09	1.83	0.50	1.00	1.00
Normalized Var2	0.86	0.60	1.91	0.69	1.50	1.00	1.00
Difference:	0.29	2.40	-1.83	1.15	-1.00	0.00	0.00
Result: 1.001760: Var1 is better than Var2							

the pair. Upon accomplishing similar comparisons for the remaining alternative pairs we shall obtain data for Table 3:

Table 3. Results of pair-wise comparisons

Alternative	Number of pair-wise comparisons won
Var1	2
Var2	1
Var3	**3**
Var4	0

Thus, the potentially best alternative is Var3, which formally appears to be the best one in the course of all pair-wise comparisons.

8. DIALOGUE DM-DSS

The elicitation of DM's preferences is carried out by comparisons of alternative pairs presented to DM beginning from the pairs wherein the PBA presumably has greater superiority.

DM-DSS dialogue can be divided into 2 stages, preparation stage and comparison stage.

Stage 1.: Preparation of comparisons.

In this stage the DM looks through the estimations of all alternatives presented to him/her and answers the following questions:

1. "Are there any estimations by one of the criteria that differ little one from another and can be considered as almost identical during pair-wise comparison?". Unification of close estimations can lead to reduced number of questions to DM.

2. "Are there any estimations that represent an inadmissibly low quality by corresponding criterion, which enables one to exclude the alternative from further consideration?". Exclusion of alternatives with noncompensible disadvantage, that is a disadvantage which can not be compensated even with the best estimations by other criteria, can also lead to reduced number of questions to DM.

The computer helps to make questions 1 and 2 more specific.

In order to prepare data for the first question, all estimations by each criterion are normalized (see formal analysis) and pairs of estimations whose normalized values have little difference (0,1÷0,15) are presented to DM for consideration.

In order to prepare data for the second question the computer selects relatively small normalized alternative estimations (0,1¸0,15) and presents them to DM.

The intervals remaining on the criteria scales after Stage 1 are compensatory. Indeed, by relinquishing these differences, DM gives himself an opportunity to compensate comparative disadvantages of an alternative by its advantages.

Stage 2. Performing multicriteria comparisons between PBA and other alternatives.

In this stage DM compares alternatives left after the 1st stage and PBA. To settle each multicriteria comparison DM is asked a series of simple psychologically correct questions. The mechanism of dividing multicriteria comparison into several simple comparisons is explained below. The method tries to split the set of criteria into several groups by 2-4 criteria so that one of alternatives in pair is better than the other for each such group. If it is possible to show that for each comparison by these groups alternative A_i is not worse than A_j, and there is at least one comparison, when A_i is better, then it is evident that A_i is better overall.

Estimations for each compared pair of alternatives are normalized and criteria are ordered by formal superiority of PBA in this pair. For example, let us consider formal comparison of Var1 and Var2 from Table 2. Pay attention to line "Difference". Positive values in this line represent advantages of Var1 in comparison with Var2. And negative values represent its disadvantages. We order these values and put them on a scale (Figure 1). Thus, on the left side of the scale there are advantages of Var1, and on the right side there are its disadvantages (or advantages of Var2). The system asks DM to compare advantages starting from the ends of the scale towards its middle. For

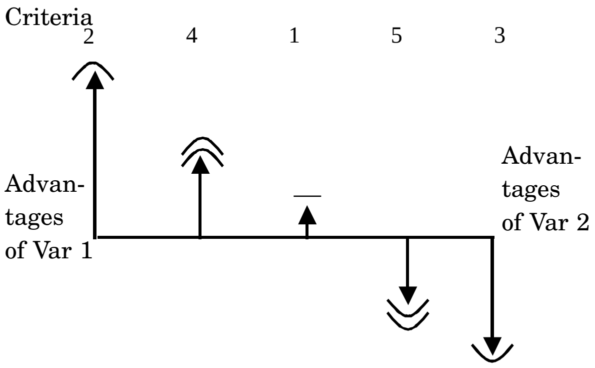


Figure 1. Scale of Normalized and Ordered Differences (SNOD)

example after a few questions to DM the first advantage beats the first disadvantage (denoted by , and the second balances the second (denoted by). After that Var1 has one more unbalanced advantage, so we can conclude that it wins the comparison.

The sequence of questions to DM is therefore determined by SNOD – Scale of Normalized Ordered Differences. The source data to form a SNOD are lines "Difference" from a formal comparison (Table 2), but DM, performing comparison, of course, pays attention to initial alternative estimations. There are relative differences between two alternatives by all criteria on the scale, beginning from advantages of one of alternatives in a pair and ending with its disadvantages. Lets note the specific features of this scale. It is:

- built for a pair of alternatives;
- relative, rather than absolute;
- ordered by objective alternative differences, rather than by DM preferences.

The constructed scale is used for a choice of questions during DM-DSS dialogue. Let us call method based on utilization of this scale SNOD (Scale of Normalized and Ordered Differences).

9. TAKING INTO ACCOUNT POSSIBLE CRITERIA INTERDEPENDENCE

In the case of criteria interdependence DM can not conclude which alternative from the pair is better considering only part of its criteria, even providing that estimations by the rest of criteria are the same. In this case DM should know those estimations. That is comparison of advantages and disadvantages of alternatives in pair should take place on definite "background" formed by estimations by the rest of criteria. And the result of alternatives comparison is dependent upon this background.

Since our method deals with comparison of given alternatives, to exclude influence of criteria dependence we propose to compare each pair of alternatives twice. The first comparison should take place on background of the first alternative, and the second (verification comparison) – on the background of the other one. If the results of both comparisons are the same then we can conclude that possible criteria dependence didn't interfere in the comparison. Otherwise it is not possible to say if one alternative is better than the other.

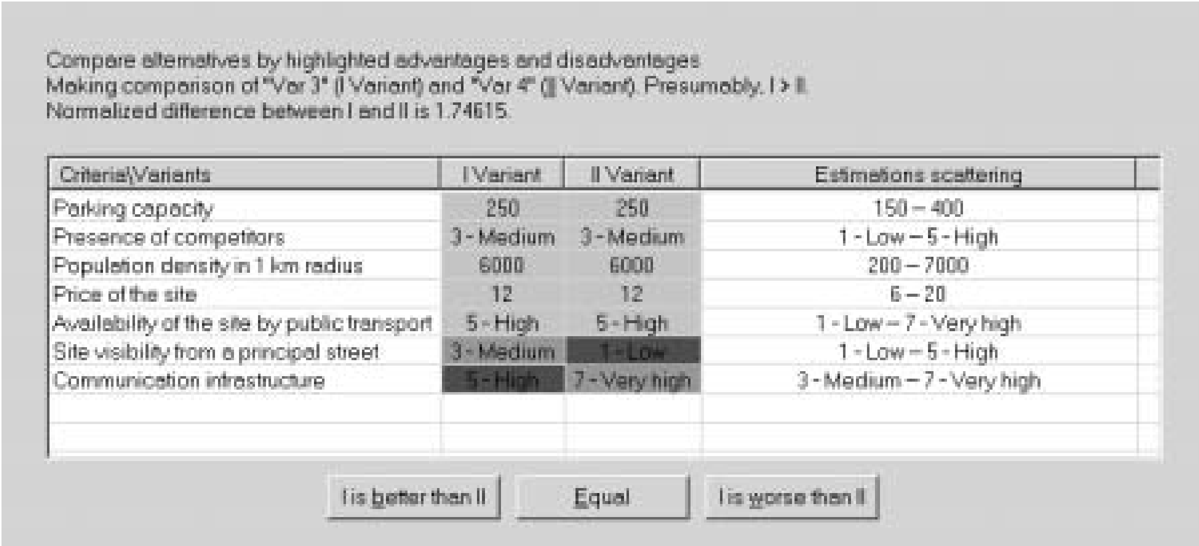
If by the results of comparison it is not possible to say that one of alternatives is better than the other then alternatives are incomparable on the basis of DM information.

10. EXAMPLE: DM-DSS DIALOGUE

Let us get back to our example. The first step is unification of close alternative estimations. The program highlights with colour the estimations that are close to each other. In our example the program advises DM to pay attention to the close estimations 250 and 300 on the first criterion (see Table 1).

Being unified these estimations could be replaced with the mean value (275) if they are approximately equally preferable for DM. This will enable one to reduce the number of questions posed.

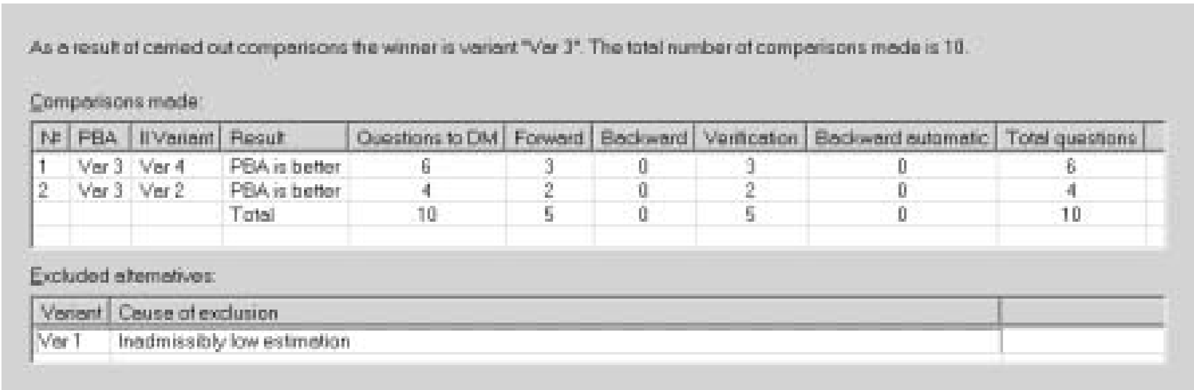
The second step includes removal of alternatives with low and non-compensatory estimations. The program will again single out low estimations by colour. In this example the program notices an extremely low estimation by the criterion "Density of population" of alternative Var1. It can be excluded from consideration if this estimation is inadmissibly low for DM.



Compare alternatives by highlighted advantages and disadvantages
Making comparison of "Var 3" (I Variant) and "Var 4" (II Variant). Presumably, I > II.
Normalized difference between I and II is 1.74615.

Criteria\Variants	I Variant	II Variant	Estimations scattering
Parking capacity	250	250	150 – 400
Presence of competitors	3 - Medium	3 - Medium	1 - Low – 5 - High
Population density in 1 km radius	6000	6000	200 – 7000
Price of the site	12	12	6 – 20
Availability of the site by public transport	5 - High	5 - High	1 - Low – 7 - Very high
Site visibility from a principal street	3 - Medium	1 - Low	1 - Low – 5 - High
Communication infrastructure	5 - High	7 - Very high	3 - Medium – 7 - Very high

Figure 2. Alternatives comparison



As a result of carried out comparisons the winner is variant "Var 3". The total number of comparisons made is 10.

Comparisons made:

№	PBA	II Variant	Result	Questions to DM	Forward	Backward	Verification	Backward automatic	Total questions
1	Var 3	Var 4	PBA is better	6	3	0	3	0	6
2	Var 3	Var 2	PBA is better	4	2	0	2	0	4
			Total	10	5	0	5	0	10

Excluded alternatives:

Variant	Cause of exclusion
Var 1	Inadmissibly low estimation

Figure 3. Results of comparisons

The third step is the main stage of DM preferences elicitation: pair comparisons using SNOD (Figure 2). Prior to this step the program determines the sequence of questions to be posed. DM is asked to compare one advantage of one alternative to one or several advantages of another alternative on different backgrounds. Again the selected estimations are highlighted with colour.

When the list of questions to DM is exhausted the results of all comparisons are presented on the computer screen (Figure 3). To establish the advantage of Var3 in our example we needed 10 questions to the DM. Alternative Var1 was excluded from the consideration due to extremely low estimation by criterion «Population density in 1 km radius».

11. CHECKING FOR INCONSISTENCIES OF DM PREFERENCES

Obtaining information from DM method checks it for consistency. Checking for consistency is realized in two stages.

1) In the stage of DM comparisons. All the comparisons dealing with particular pair of criteria are checked for consistency. If there are contradictory comparisons then the method checks if the contradiction is caused by criteria interdependence, which was already taken into account by DM. If the contradiction can not be explained by criteria interdependence then contradictory comparisons are presented to DM for analysis and elimination of contradictions.

2) In the stage of multicriteria alternatives comparisons.

The verification is accomplished by memorizing of all pair comparisons done by DM. If there are cycles on alternatives set, DM is presented all pair comparisons for alternatives forming a cycle and requested to reconsider part of them to remove the cycle.

12. GETTING RECOMMENDATIONS

Thus, method SNOD allows to help DM choosing the best alternative in many cases. However, it doesn't always lead to this result, alternatives may turn out to be incomparable.

In this case method gives DM the following information:

1. Points out an alternative with the maximum sum of normalized estimations as presumably best one.
2. Informs that there is another alternative quite close to the best one and lists their estimations.
3. Suggests to introduce additional criteria, by which it is possible to distinguish incomparable (by this method) alternatives.

13. STATISTICAL MODELING

At the heart of proposed method there is multicriteria alternatives comparison utilizing Scale of Normalized and Ordered Differences. It is reasonable to question how effective this comparison is, that is how many questions are posed to DM, and how often there can be incomparability of alternatives.

In fact, the comparison of alternatives consists in an attempt to cover disadvantages of an alternative by its advantages. To illustrate this statement let's get back to Figure 1. Suppose that the length of vertical arrows (advantages and disadvantages) mean their real value to DM. Then covering disadvantages by advantages literary means covering negative arrows (disadvantages) by positive ones (advantages) (Figure 4).

This covering can be optimally built using full enumeration of possibilities algorithm. But the main difficulty is that we do not know in advance how to compare arrow lengths (that is values of advantages and disadvantages). To compare them we need to pose a question to DM. And an algorithm of full enumeration of possibilities could require too many comparisons.

Construction of covering by method SNOD is performed with linear speed (that is number of questions to DM is proportional to the number of advantages and disadvantages in alternatives pair). However it doesn't result in optimal covering.

Besides, advantages and disadvantages on the difference scale are ordered formally, not by their value to DM, which can also influence the optimality of covering.

To evaluate the above statements quantitatively we carried out statistical modeling. As a model of DM it was used very simple one realizing value function $V(y) = y$. In other words, the length of arrows was taken as real value to DM.

For a given number of advantages and disadvantages (both were from 1 to 7) scale of

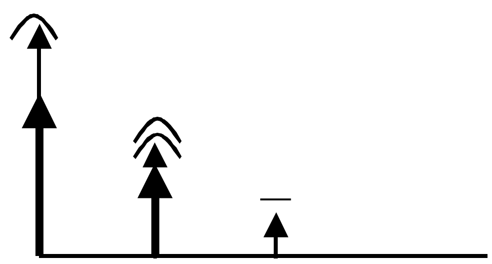


Figure 4. Covering

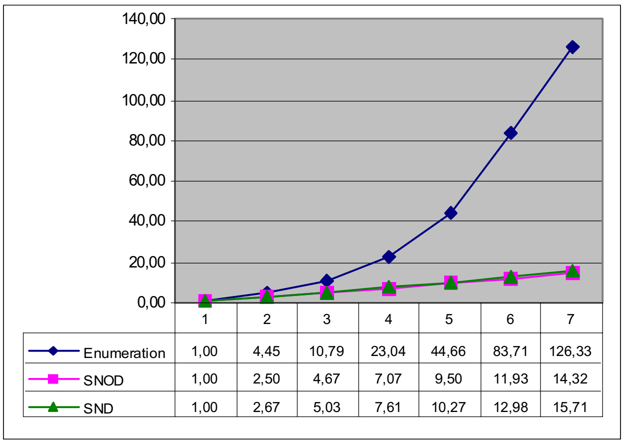


Figure 5. Number of questions to DM

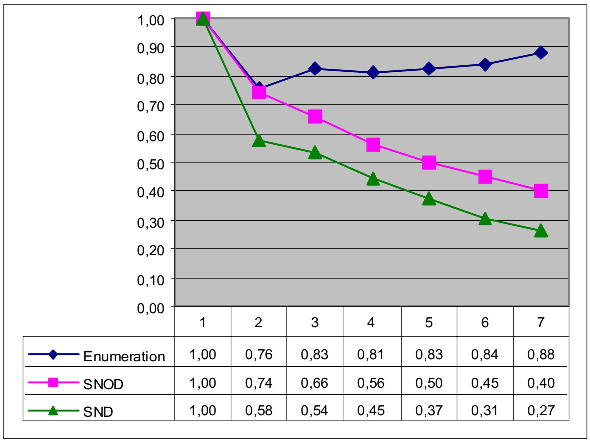


Figure 6. Percentage of comparable alternatives

differences was generated randomly 10000 times. Then comparison was made by three algorithms:

1. Algorithm of full enumeration of possibilities (Enumeration)
2. Algorithm SNOD with unordered scale (SND)
3. Algorithm SNOD with ordered scale (SNOD)

For each case number of questions posed to DM and the result of comparison (comparable or incomparable) were memorized. The results are tables 7x7, where for each pair <number of advantages, number of disadvantages> there is average number of questions to DM and percentage of comparable cases. However, for the sake of clearness we present here only the results of comparisons with the same number of advantages and disadvantages (Figure 5 and Figure 6).

On the X axis there are numbers of advantages and disadvantages in compared alternatives pair.

As it can be seen, in the case of full enumeration of possibilities number of questions to DM is quite large and increases exponentially with the growth of alternatives dimension. Algorithms SNOD and SND ask number of questions, which linearly depends upon the dimension, and this fact allows their practical usage.

Percentage of comparable alternatives for algorithm of full enumeration of possibilities is maximum and can not be overcome by any other algorithm. The figure shows that it is high enough and fluctuates in the range 0,8÷0,9. Algorithms SNOD and SND shows lesser effectiveness: percentage of comparable alternatives for them lowers with growth of dimension and amounts to 0,4÷0,8 for SNOD and 0,3÷0,6 for SND. In practical applications of the algorithm SNOD differences are ordered formally, which often doesn't coincide with DM opinion, but only correlates with it. That's why real percentage of comparable alternatives for algorithm SNOD will lay somewhere between the curves SNOD and SND.

In conclusion one can note, that effectiveness of the method SNOD can be improved using more complex algorithm in case of incomparability (for example, full enumeration of possibilities), which can resolve the incomparability at the cost of additional questions to DM.

14. CONCLUSION

The method SNOD proposed to select the best alternative from a group of given alternatives has the following features, common for all methods from Verbal Decision Analysis group:

1. The method uses rather simple (from the psychological point of view) procedures for the elicitation of DM preferences.
2. The language of DM and his organization is used in DM-DSS dialogue. The quantitative, verbal and mark estimations may be used.
3. The method enables one to compare a large number of alternatives by using the minimum number of questions to DM.
4. The method can easily adapt to a concrete problem (set of alternatives) and always results in the best (or presumably best) alternative.
5. The method checks the consistency of information obtained from DM.
6. The method provides DM with an opportunity to receive explanations of the results by displaying those of his/her answers, which have led to the result obtained.

On the basis of our experience we could conclude that method SNOD is convenient for DM, "transparent" and effective method for solution of a great variety of tasks.

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