Ranking Multicriteria Alternatives: the method ZAPROS III

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Abstract: The new version of the method for the construction of partial order on the set of multicriteria alternatives is presented. This method belongs to the family of Verbal Decision Analysis methods and gives a more efficient means of problem solution. The method is based on psychologically valid operations for information elicitation from a decision maker: comparisons of two distances between the evaluations on the ordinal scales of two criteria. The information received from a decision maker is used for the construction of a binary relation between a pair of alternatives which yields preference, indifference and incomparability relations. The method allows construction of a partial order on the set of given alternatives as well as on the set of all possible alternatives. The illustrative example is given.

Keywords: Decision theory; Multiple criteria; Behavior; Project management

1. Introduction

The problems of ranking alternatives having estimations on multiple criteria are widespread in human life. There are many decision aiding methods oriented to the solution of the problems [1,2,3,4].

In this paper we discuss the multicriteria ranking problem having the following features:

1) A decision rule is to be developed before appearance of alternatives;

2) There are a large number of alternatives;

3) Evaluations of alternatives upon criteria could be given only by human beings playing the role of measurement devices.

4) The quality grades on criteria scales are verbal definitions presenting subjective values of the decision maker.

The method ZAPROS (abbreviation of Russian words: Closed Procedures near Reference Situations) has been developed for the solution of problems having such features. This method belongs to the family of Verbal Decision Analysis (VDA) methods [4,10].

VDA methods are oriented to the solution of problems of a special kind.

Let us stress the common features of such problems:

1. Factors in these problems are of purely qualitative, subjective nature, especially difficult for formalization and numerical measurement (prestige of an organization, attractiveness of a dress, attitude towards reforms, etc.); the factors are usually described in language accepted by the decision maker;
2. The process of task analysis is also subjective by nature: rules for consideration and comparison of the main qualitative factors are mainly defined by the decision maker.

Therefore, the decision maker is the key element of the problem. This must be recognized, and attention must be paid to the capabilities and limitations of the human information processing system and to the results of investigations on human errors and heuristics [5].

The problems having such features have been called unstructured [6].

The first version of ZAPROS was published in 1978 [7]. The second version [8,9] gives the development of the original ideas. Both versions are based on the similar procedures of information elicitation from the decision maker.

The method ZAPROS-III presented in this paper uses the preference elicitation procedure proposed in first version of the method [7]. But it has quite a different structure to give more rational and strict justification to the method:

1. A simpler and more transparent procedure for the construction of a joint ordinal scale for quality variation along the scales of criteria is used.
2. New justification is given for the procedure of alternatives comparison
3. The method gives the absolute as well as relative (as in the previous versions) ranks of alternatives.

2. Example

The practical problem is to organize a fund for investing money in R&D projects. The fund organizer is interested in developing an effective system for selection of the best projects. A decision analyst is used to carry out the job. It is decided that highly qualified experts are to be involved in the process of project estimation.

The fund organizer (we will call him the decision-maker) in cooperation with the analyst develop a list of the most important criteria for project evaluation. The list of these criteria with possible values on their scales is:

Table 1. Criteria and possible values for evaluation of R&D projects

Criteria	Possible values on their scales
A. Originality	A1. Absolutely new idea and/or approach
	A2. There are new elements in the proposal
	A3. Further development of previous ideas
B. Prospects	B1. High probability of success
	B2. Success is rather probable
	B3. Success is hardly probable
C. Qualification	C1. Qualification of the applicant is high
	C2. Qualification of the applicant is normal
	C3. Qualification of the applicant is unknown

It is easy to note that criterion values are given from the most preferred to the least preferred (according to the preferences of the decision maker).

The projects to be submitted to the fund are not known in advance. It is necessary to rank-order the submitted projects according to their overall value. Each project requires some resources. Given a ranking of projects it is easy to select a group of best projects within the limit of available resources.

The question is: how to construct a rank-order for all possible combinations of the evaluations upon the criteria (in our case 27 combinations) on the basis of the decision maker preferences?

3. **Formal statement of problem.**

The problem may be formulated as follows:

Given:

1. $K = 1,2,...,N$ is a set of criteria;
2. n_q is the number of possible values on the scale of the q-th criterion $(q \in K)$;
3. $X_q = \{x_{iq}\}$ is a set of values for the q-th criterion (the scale of the q-th criterion); $|X_q| = n_q$ $(q \in K)$; the values on a scale are ordered from best (first) to worst (last); the order of the values on one scale does not depend upon values on the others ;
4. $Y = X_1 * X_2 * \dots X_N$ is a set of vectors $y_i \in Y$ of the following type $y_i = (y_{i1}, y_{i2}, \dots, y_{iN})$ where $y_{iq} \in X_q$ and $P = |Y| = \prod_{i=1}^{i=N} n_i$;
5. $A = \{a_i\} \in Y$; $i = 1,2....t$ -the set of t vectors describing real alternatives.

Required:

to rank multicriteria alternatives on the basis of a decision-maker's preferences.

4. **Elicitation of DM's preferences**

4.1. Joint scale of quality variation for two criteria

Let us look at the criteria list and assume that we have an “ideal” object, assigned all the best values on all criteria. We usually do not have such an alternative in real life. We will use this ideal alternative as a “reference situation”. Deviating from this ideal, we will lessen the quality of the hypothetical object on two criteria.

Let us introduce the notion of quality variation (QV). Quality variation is the result of changing one evaluation on the scale of one criteria.

The task of decision maker preference elicitation consists in pair-wise comparison of all QV taken from the scales of two criteria, by supposition that there are the best evaluations on the other criteria. Evidently, QV along a criterion scale is equal to the sum of QV between the evaluations on the same scale.

The typical question posed to the decision maker is:

"What do you prefer for the transfers from better evaluations to worse ones:

$x_{if} \Rightarrow x_{ik}$ *or* $x_{js} \Rightarrow x_{ju}$ (k>f; u>s) ? "

The possible decision maker answers are: «the first» or «the second» or «they are equal for me».

Decision maker responses allow ranking of all QV from the scales of two criteria. This ranking could be called the Joint Scale of Quality Variation (JSQV) for two criteria.

Next, a different pair of criteria is taken for QV comparison by supposition that the evaluations for other are best.

There are 0.5N(N-1) possible pairs of criteria. The preferences of the decision maker are elicited for each pair. So, 0.5N(N-1) rankings of QV for all pairs of criteria could be constructed .

Let us illustrate the method by the example.

Let us look at the criteria list and assume that we have an «ideal» project, assigned all the best values on all criteria. We will lessen the quality of the hypothetical project against two criteria: A («Originality») and B («Prospects») criteria having the best evaluations on C. In the example we have three QV for each criterion. Therefore, the task for the decision maker consists in pair-wise comparison of six QV. Not all comparisons are needed: QV along the scales are equal to the sum of QV between the evaluations on the same scale. But, it is possible to use the transitive closure: for example, if

$b_3 > b_2$ *and* $b_2 > a_2$ *than* $b_3 > a_2$.

$b_3 > b_2$ *and* $b_2 > a_2$ *than* $b_3 > a_2$

Eight comparisons are needed to rank QV's from the scales of A and B (4 relations follow from transitivity).

Let us introduce the following notions for QV's:

$A_1 \Rightarrow A_2 = a_1$; $A_2 \Rightarrow A_3 = a_2$; $A_1 \Rightarrow A_3 = a_3$; $B_1 \Rightarrow B_2 = b_1$; $B_2 \Rightarrow B_3 = b_2$; $B_1 \Rightarrow B_3 = b_3$;

The generic question to DM is:

Question. "What do you prefer: the transfer from a project with an absolutely new idea to one with new elements in the proposal or the transfer from a project but with high probability of success to one where the success is rather probable?"

Posing the questions in a similar way one could rank QV. Let us suppose that DM preferences define the following ranking:

$$a_1 \langle\, b_1 \,\langle\, a_2 \,\langle\, b_2 \,\langle\, a_3 \,\langle\, b_3$$

This ranking is JSQV for the criteria A and B by presupposition that it is the best evaluation on criterion C.

In the same way JSQV for the pairs (A and C) and (B and C) can be constructed.

4.2. The check of independence for two criteria.

JSQV for a pair of criteria holds valuable information about DM preferences. But the possibility of its utilization depends on the independence of the decision maker comparisons with a "reference situation".

Definition 1.

Let us call two criteria *independent by quality variation* if JSQV constructed for those criteria do not depend from the evaluations on other criteria.

The check of independence by QV is.

Let us take a quite different "reference situation"-the worst evaluations on all criteria. It is possible to compare QV on the scale of two criteria and check the correspondence to the ranking made near first "reference situation".

Only some part of QV comparisons could be taken with the aim of testing the independence condition.If there is no difference in QV comparisons for the same pair of criteria near different "reference situations" we could accept that the two criteria are independent by QV.

Two “reference situations” are quite contrasting alternatives. It is possible to accept that the condition of independence is true if such “reference situations” have no influence on the comparisons made by the decision maker.

For the example given above it is necessary to repeat some comparisons of QV by supposition about the evaluation C3 on criterion C.

Let us suppose that the results of the comparisons are:

$b_3 > a_3 > b_2 > a_2$.

$b_3 > a_3 > b_2 > a_2$.

In this case we could accept the independence of criteria A and B.

For cases when a pair of criteria are not independent by QV, it is necessary to change the verbal description of a problem and achieve independence (see the examples in [4]).

Let us note that the condition of independence by QV is close to the condition of preference independence [1].

4.3. Independence for the group of criteria.

The dependence of a pair of criteria on the rest of the criteria is the best-understood case. It might be well to note that this kind of (in)dependence of criteria is checked in many decision methods. It was proven that, if all criteria are pair-wise preference independent , any group of criteria is independent of the rest of criteria [1].

We refer to the opinion of D. von Winterfeldt and G. Fisher [11] that the group dependence of criteria «is indefinite in nature and difficult to detect» if the criteria are pair-wise independent. Really, one could not find such dependence in practical cases.

Consequence

In the case when all pairs of criteria are independent by QV it is possible to accept that all criteria are independent by QV.

The test for independence is sufficient because all pairs of criteria are considered. Therefore, one could take any pair of criteria independently from the others to analyze the differences in evaluations of alternatives.

4.4. Joint Scale of quality variation for all criteria.

On the basis of information elicited from the decision maker for each pair of criteria it is possible to construct JSQV for all criteria. The noncontradictory rankings of QV from all criteria scales are compared many times. It is necessary to find the place of each QV on the joint scale.

For the construction of the joint scale for all criteria it is possible to use the following algorithm:

Sequential selection of non-dominating QV.

Joint Scales for the pairs of criteria could be taken as the graphs having the same root: zero quality decreasing. Let us take the node of this joint graph that is not dominated by any other and put it on the joint scale. After excluding such nodes from all graphs, let us find the next non-dominated node, put it on the joint scale and so on. It is easy to see that such algorithm gives the rank-order of all QV .

Let us take the example given above.

Additional to the JSQV given above for criteria A and B, it is possible to construct scales of such kind for the pairs (A and C) and (B and C). Let us suppose that we have the following results.

$c_1 \langle a_1 \langle a_2 \langle c_2 \langle a_3 \langle c_3$ and $c_1 \langle b_1 \langle b_2 \langle c_2 \langle b_3 \langle c_3$

Using the algorithm of sequential selection of non-dominated QV, it is possible to construct the joint scale of QV for all criteria:

$c_1 \langle a_1 \langle b_1 \langle a_2 \langle b_2 \langle c_2 \langle a_3 \langle b_3 \langle c_3$

4.5. The check of information for contradictions.

In the process of construction of the JSQV for two criteria, it is easy for the decision maker to check comparisons for possible contradictions. That is not the case for the construction of JSQV for all criteria.

Certainly, people could make errors. Therefore, we need special procedures for finding and eliminating human errors.

Fortunately, a special, “closed” procedure for finding and eliminating the decision maker contradictions has been proposed [7].

By constructing JSQV for every pair of criteria we would require additional data from the decision maker. The additional information is used to create the check for consistency. If on some step of the algorithm for sequential selection of QV it is not possible to find the next non-dominated QV, there is a contradiction in the decision maker preferences. The algorithm can discover the contradictory answer and demonstrate them to the decision maker for correction.

Let us return to the example presented above.

Let us suppose that instead of JSQV for criteria C and B given above, we have the following:

$b_1 \langle c_1 \langle b_2 \langle c_2 \langle b_3 \langle c_3$

In this case, when combining three scales into one, there is the contradiction:

$b_1 \langle c_1 \langle a_1 \langle b_1$

The contradiction does not allow finding a place of corresponding QV on the joint scale. Usually such a contradiction is the result of an irrational judgment. It is necessary for the decision maker to analyse the situation and find a rational compromise.

This contradiction is to be presented to the decision maker for analysis and resolution.

The construction of Joint Scale of Criteria Variations gives a check of the decision maker input for contradiction. The possibility to combine pair-wise scale into a JSCV confirms the absence of contradictions in decision maker judgments.

The questions needed for JSCV construction represent the dialog between the decision maker and the computer.

4.6. Special case.

For N=2, the meaning of “reference situations” disappears. For the remaining two criteria it is possible to construct a JSQV and use it for alternative comparison (see below).

4.7. Psychological basis of the procedure.

The procedure of decision maker preference elicitation proposed above is justified from a psychological point of view. All questions to the decision maker are formulated in natural language, in terms of verbal evaluations on criteria scales. The kind of questions(comparison of two quality variations) is psychologically valid [10].

The proposed procedure of DM preference elicitation was checked in experiments with a group of subjects [7]. For 5 to 7 criteria with 2 to 5 evaluations on criteria scales the number of contradictions was 1 to 3. When subjects were presented the contradictions, they removed them to construct a consistent decision rule.

5. **Comparison of alternatives.**

5.1. Comparison of two alternatives.

Statement 1.

The quality of every alternative can be expressed as the vector of QV corresponding to the evaluations of the alternative upon the criteria.

Proof.

Each evaluation of an alternative is connected with some QV. In the case of independence by quality variation, it is possible to represent the quality of an alternative by the set of QV, each of them corresponds to the distance along a scale of one criterion between the evaluations. Therefore, the vector QV represents the quality of an alternative.

Statement 2

The relation between any pair of QV on JSQV is defined or determined by direct answers of DM or on the basis of expansion by transitivity.

The proof.

Let us take two arbitrary QV from JSQV. It is possible to find JSQV for the pair of criteria or for one criterion that they belong to. For both cases those two QV are compared or assessed directly by the decision maker or by utilization of the transitivity condition.

Definition 2.

Let us note as the function of alternative quality: $V(y)$.

Let us make the following supposition about the properties of this function:

-There are maximum and minimum values of $V(y)$;

-For independent criteria, the value of $V(y)$ is increasing when the evaluation on each criteria are improving.

Let us assign a rank for each QV on JSQV beginning from the best QV.

For example, for JSQV given above,

$c_1 \langle\ a_1 \langle\ b_1 \langle\ a_2 \langle\ b_2 \langle c_2 \langle\ a_3 \langle\ b_3 \langle\ c_3$

rank 1 is given to c_1, rank 2 is given to a_1 and so on.

Let us take two alternatives: $y_i = (y_{i1}, y_{i2}, \ldots, y_{iN})$ and $y_j = (y_{j1}, y_{j2}, \ldots, y_{jN})$.

It is possible to find a corresponding QV for each component of vectors and rank each QV according to JSQV.

For each alternative it is possible to define the corresponding vector of components ranks:

$V(y_i) \Leftrightarrow V(r_k, r_t, \ldots, r_g)$

$V(y_j) \Leftrightarrow V(q_s, q_d, \ldots, q_m)$

where:

$r_k, r_l, \ldots r_g$ - ranks of components for the vector $y_i = (y_{i1}, y_{i2}, \ldots, y_{iN})$.

$q_s, q_d, \ldots, q_m$ - ranks of components for the vector $y_j = (y_{j1}, y_{j2}, \ldots, y_{jN})$.

<u>Statement 3.</u>

If the condition of independence by QV is true for all pairs of criteria and ranks of the components for y_i are no worse than the ranks of the components for y_j and at least for one components of y_i rank is better, than alternative y_i is more preferable for the decision maker in the comparison with y_j, and $V(y_i) \succ V(y_j)$.

Proof.

If the condition of independence by QV is true, from the replacement of one component of y_i by one component of y_j, it follows:

$V(y_i) = V(r_k, r_t, \ldots, r_g) \geq\ V(q_s, r_l, \ldots, r_g)$

Making the replacement by one of the components of y_i by the components of y_j, we have the following inequalities:

$V(q_s, r_t, \ldots, r_g) \geq\ V(q_s, q_d, \ldots, r_g)$

……………………………….

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$V(q_s, q_d, \ldots, r_g) \geq\ V(q_s, q_d, \ldots, q_m) = V(y_j)$

Making the sums of left and right sides of inequalities, one has:

$V(y_i) \succ V(y_j)$.

The following two statements are evident.

Statement 4.

If the components of both vectors have the same ranks, the vectors are equivalent.

Statement 5.

If the conditions of the statements 3 and 4 are not true, the alternatives y_i and y_j are incomparable.

The comparison of two vectors on the basis of JSQV makes it possible to demonstrate the preference of one alternative against the other or their equivalence. If such information is not sufficient, the alternatives are incomparable.

5.2. Ranking of given alternatives

Let us suppose that the group of alternatives is given. The construction of a partial order of alternatives is completed by the following three-step algorithm.

Step 1: Formal index of quality

A formal index of quality is introduced to minimize the number of pair-wise alternative comparisons needed for partial ranking of alternatives.

For each alternative it is possible to make the sum of corresponding ranks for QV of its components. Let us nominate this sum as a Formal Index of Quality (FIQ). It is evident that a better alternative in a pair-wise comparison always has a smaller FIQ.

Let us return to the example given above. For general JSQV for three criteria it is possible to assign the following numbers to QV:

$c_1 \langle a_1 \langle b_1 \langle a_2 \langle b_2 \langle c_2 \langle a_3 \langle b_3 \langle c_3$

1 2 3 4 5 6 7 8 9

The index is calculated formally, even with the violation of properties of ordinal scales: the sum of QV between the evaluations on one criterion scale is not equal to QV along the scale. But we need this index only for finding the sequence of pair-wise alternative comparisons.

Step 2 : The procedure of pair-wise comparisons.

The following procedure of comparison is used.

1) The alternatives are allocated according FIQ from smaller sums to bigger ones. It is easy to notice that potentially best alternatives have smaller sums.

2) According to this order, alternatives are compared pair-wise on the basis of binary relation given in Statement 4 beginning from an alternative with smallest FIQ.

3) Let us take three alternatives with increasing FIQ. If in the pair-wise comparisons one could find that:

$Alt\ i \Rightarrow Alt\ j \Rightarrow Alt\ k$, then from definition of JSQD follow :

$Alt\ i \Rightarrow Alt\ k$.

If $Alt\ j\ and\ Alt\ k$ are incomparable, then $Alt\ i$ is to be compared with both alternatives.

<u>Step 3 : Sequential selection of non-dominated nuclei</u>

Let us conduct pair-wise comparison of alternatives beginning from ones with smaller FIQ on the basis of binary relation formulated in Statement 3.

Let us single out, on the basis of binary relations all nondominated alternatives and refer to them as the first nucleus. After removing the first nucleus, let us select the second, and so on.

The rule of rank assignment to an alternative is the following. Alternatives from the first nucleus have rank 1. An alternative is ranked «i» if it is dominated by an alternative ranked « i-1» and itself dominates an alternative ranked «i+1».

If an alternative is dominated by an alternative ranked «i» but is itself dominates an alternative ranked «i+j» ,then its rank is fuzzy within the range from «i+1» to «i+j-1».

<u>Relative versus absolute ranks.</u>

The ranks nominated in the way defined above could be called relative because they are related to the given group of alternatives. But using the same three-step algorithm it is possible to assign absolute rank to every alternative. The ranks can be called absolute if they are related to all possible alternatives from set Y (all possible combinations of evaluations upon the criteria). To find absolute ranks, it is necessary to apply the three-step algorithm for the set of vectors $y_i \in Y$.

Let us return to the example given above and assign to every QV on general JSQV ranks beginning from 1 for the smallest. In table 2 the group of 9 alternatives is given with corresponding FIQ.

Let us demonstrate how FIQ is calculated. For example, for Alt 3 QV are a_3 and c_1; the sum is equal 8. For Alt 4 the only QV is b_3; the sum is equal 8.

Table 2.

N of alternative	Evaluations on criteria	FIQ
Alt 1	A1 B2 C2	4
Alt 2	A2 B2 C1	5
Alt 3	A3 B1 C2	8
Alt 4	A1 B3 C1	8
Alt 5	A2 B1 C3	11
Alt 6	A3 B2 C2	11
Alt 7	A3 B2 C1	10
Alt 8	A3 B3 C1	15
Alt 9	A2 B1 C2	3

On Fig. 1 below the partial order of the alternatives is given as the result of binary comparisons. The alternatives are located from best (on the left) to worst. The alternatives in pairs: Alt 2-Alt 3, Alt 3-Alt 4, Alt 5-Alt 7, Alt 5-Alt 6, and Alt 5-Alt 8 are incomparable.

Let us demonstrate how the comparisons are made. First, the comparison of Alt 9 and Alt 1 is done. It corresponds to the comparison of a_1 and b_1 on general JSQV. According to JSQV (see above) Alt 9 is better.

Then, Alt 1 is compared with 2 and 3 in the accordance with FIG values. It is not necessary to compare Alt 9 with Alt 2 and Alt 3 because they are dominated by Alt 1. Really, c_1 is less than a_1, and b_1 is less than a_3 on general JSQV. The next alternative involved in the process of the comparison is Alt 4 (FIQ=8) and so on. Let us take Alt 5 and 7. In pair-wise comparison one needs to compare c_3 with a_2 and b_1. We do not have enough information on JSQV to make such a comparison.

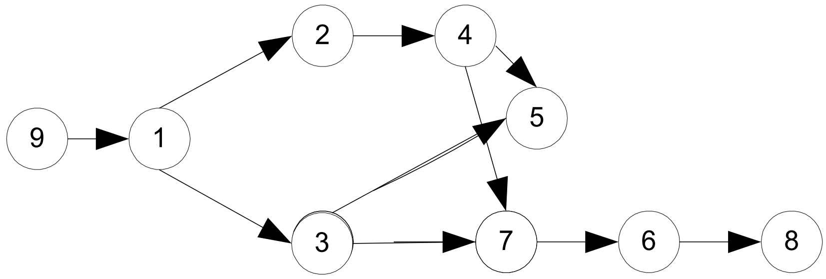

Fig. 1. The partial order of alternatives.

Let us note that only 15 binary comparisons are made to obtain the partial order of the alternatives presented on Fig. 1 instead of 36 required by the previous version of ZAPROS.

Using the rule of rank assignment given above the alternatives receive the ranks shown in Table 3.

Table 3.

N of Alter	Alt 9	Alt 1	Alt 2	Alt 3	Alt 4	Alt 7	Alt 5	Alt 6	Alt 8
Rank	1	2	3	3-4	4	5	5-7	6	7

The ranks in Table 3 are relative and reflect the relative values of the alternatives. Taking all vectors from the set Y it is possible to receive the absolute ranks of alternatives. For example, Alt 9, the best of the group, has in general ranking of the alternatives from the set Y the rank 4 that gives the indication of the possible absolute value of this alternative.

Therefore, the decision maker can have relative as well as absolute ranks of alternatives. The decision maker could allocate the resources using relative ranks but taking into account the absolute values of available alternatives to make (if necessary) the correction of resources distribution.

In the general case, the absolute ranks correspond to the general aim of ZAPROS: to prepare a decision rule before the appearance of alternatives. The three-step algorithm given above allows one to achieve this goal.

6. **Conclusion.**

The important feature of ZAPROS methods is the utilization of psychologically justified ways of a decision maker preference elicitation. Such an approach takes into account the possibilities and limitations of human information processing system.

In a comparison with MAUT approach [1] the output of ZAPROS is very approximate. Some alternatives could be incomparable. Alternatives have only ranks (sometimes fuzzy) instead of exact quantitative evaluations of utility.

But such approximate output is much more reliable. A decision maker could use ZAPROS to gradually develop a consistent and noncontradictory policy. In experiments [12] it was demonstrated that the methods based on MAUT are very sensitive to small human errors. Such errors are inevitable because human beings are not exact measurement devices producing exact quantitative measurements.

On the other hand, the relations between alternatives received as the output of ZAPROS are stable [12].

As the conclusion, it is possible to say:

Better to be approximately right than exactly wrong.

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References.

1. Keeney R. L., Raiffa H., *Decisions With Multiple Objectives: Preferences And Value Tradeoffs*, New York, Wiley, 1976.
2. Saaty T.L., *The Analytic Hierarchy Process*, New York, McGraw-Hill, 1980.
3. Roy B. *Multicriteria Methodology for Decision Aiding*, Kluwer Academic Publisher, Dordrecht, 1996.
4. Larichev O.I., Moshkovich H.M. *Verbal Decision Analysis For Unstructured Problems.,* Boston, Kluwer Academic Publishers, 1997.
5. Kahneman D., Slovic P., Tversky A. (*Eds.*), *Judgement Under Uncertainty: Heuristics And Biases.*, Cambridge University Press, Cambridge, 1982.

6. Simon, H., Newell A. Heuristic Problem solving: the next advance in operations research., *Operations Research*, v.6, 1958.

7.Larichev, O. I., Yu. A. Zuev, L. S. Gnedenko: Method ZAPROS (CLosed Procedures near Reference Situations) for the analysis of variants of complex decisions, In: S. V. Emelyanov (eds.), *Multicriteria Choice for the Solution of Ill-structured Problems,* Moscow: VNIISI Proceedings, N 5 (in Russian), 1978, pp. 83-97.

8. Gnedenko L.S., Larichev O.I., Moshkovich H.M., Furems E.M. Procedure for Construction of a Quasi-Order on the Set of Multiattribute Alternatives on the Basis of Reliable Information about the Decision Maker's Preferences. *Automatica i Telemechanica*, N 9, 1986,pp 104-113 (in Russian).

9. Larichev O. I., Moshkovich H. M., ZAPROS-LM-A Method And System For Rank-Ordering Of Multiattribute Alternatives*., European Journal Of Operations Research*, V. 82, 1995, Pp. 503-521.

10. Larichev O. I., Cognitive Validity In Design Of Decision-Aiding Techniques*, Journal of Multi-Criteria Decision Analysis*, V. 1, N 3, 1992, Pp. 127-138.

11. von Winterfeldt, D., G.W. Fischer: Multiattribute utility theory: Models and assessment procedures, In: D. Wendt, C. Vlek (eds.) *Utility, probability and human decision making,* Dordrecht, Reidel, 1975.

12. Larichev O. I., Olson D. L., Moshkovich H. M., Mechitov A. I., Numerical Vs. Cardinal Measurements In Multiattribute Decision Making: How Exact Is Exact Enough? *Organizational Behavior And Human Decision Processes*, 64, 1995, 1, 9-21.

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