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MULTI-STAGE TECHNIQUE 'PAKS' FOR MULTIPLE CRITERIA DECISION AIDING

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This paper describes a new multi-stage technique PAKS (the abbreviation of Russian words: Consequent Aggregation of Classified Situations) for sorting multi-attribute objects by a group. Based on decision maker preferences, this technique provides a hierarchical aggregation of a large number of initial attributes into a smaller number of criteria, thus reducing the dimension of the attribute space progressively, using various tools of verbal decision analysis. The consequent aggregation of attributes allows generating manifold lists of composite criteria with numerical and/or verbal scales, finding and analyzing the most preferable solution of a decision problem. The technique was applied to evaluate efficiency of R&D projects, which were assessed by several experts on many qualitative criteria and subsidized by the Russian Foundation for Basic Research.

Keywords: Decision aiding; group sorting; multi-attribute objects; verbal decision analysis; hierarchical aggregation; reduction of attribute space; composite criterion; efficiency.

1991 Mathematics Subject Classification: 90A05, 90A07, 90B50

1. Introduction

Sorting objects into several classes by their properties is a typical problem in multiple criteria decision aiding (MCDA), operational research, pattern recognition, and other areas. In real-life situations, objects are characterized by many diverse attributes, number of which may be very large. Such problems, for example, include a competitive selection of R&D projects[1] and an evaluation of credibility of credit card holders.[2] Directly comparing and sorting alternatives presented with numerous attributes is a rather difficult procedure for a decision maker (DM) and needs special decision aiding techniques.

While building a method for classification, it is important to take into account the cognitive possibilities of a DM. According to psychological experiments,[3] people use simple strategies to sort alternatives. For instance, they use only a subset of criteria if the number of criteria is more than 5, the number of grades on a criteria scale is more than 4, and the number of decision classes is more than 5. The use of various simplified strategies that address only a part of the available information affects negatively solving the problem of multicriteria choice and complicates further analysis of the results. Hence the DM needs tools for multicriteria choice in spaces of higher dimension. One of the helpful approaches to overcome these difficulties is to reduce the dimension of the attribute space. Special studies have shown that people easily compare objects represented by a small number of indicators, and made fewer errors. The results of such operations are more reliable, more understandable.

In this paper, we consider a new multi-stage technique PAKS (the abbreviation of Russian words: Consequent Aggregation of Classified Situations) for MCDA. This technique provides sorting of multi-attribute objects on a composite criterion and depends on DM preferences. The interactive procedure includes reducing the dimension of the multi-attribute space and forming the aggregated classification rules with various methods of verbal decision analysis. Thus, a large number of initial attributes are combined step by step into a smaller number of composite criteria with verbal scales. Grades of the top level composite criterion (complex indicator) correspond to the given classes. Values of grades of composite criteria are used to compare and sort multi-attribute objects. The suggested technique has been applied to evaluate efficiency of R&D projects, which were estimated by several experts on many qualitative criteria and subsidized by the Russian Foundation for Basic Research.

2. Overview of MCDA Methods

The problem of multicriteria ordinal classification is formulated as follows. A given set of alternatives $A_1, \dots, A_p$ is assessed using many criteria $K_1, \dots, K_m$. Each criterion K_i has an ordered discrete scale $X_i = \{x_i^1, \dots, x_i^{gi}\}$, $i = 1, \dots, m$. Based on DM preferences, it is required to assign the alternative to one of the given classes (categories) $D_1, \dots, D_l$.

Usually a multi-attribute object A_q is represented as a tuple (vector or cortege) $x_q = (x_{q1}^{e1}, \dots, x_{qm}^{em})$ in the Cartesian space $X_1 \times \cdots \times X_m$ of numerical or verbal attributes. If one and the same object A_i is evaluated k times (for instance, by k experts or with k methods), then this object may be represented as a group of k vectors/corteges $\{x_q^{(1)}, \dots, x_q^{(k)}\}$. Here $x_q^{(f)} = (x_{q1}^{e1(f)}, \dots, x_{qm}^{em(f)}), f = 1, \dots, k$ is a fth copy of the object A_q. The group of tuples is to be considered as a whole in spite of a possible incomparability of separate tuples $x_q^{(f)}$. This complex group has an overcomplicated structure that is very difficult to analyze.

Typically, in order to simplify such a situation, a group of k vectors $\{x_q^{(1)}, \dots, x_q^{(k)}\}$ with numerical components is replaced by a single vector.[4,5] For example, this single vector can have components derived by averaging or weighting the attribute values of all the group members. This single vector can be a center of the group or the closest to all vectors within the group. However, note that the properties of all objects in a group may be lost after such a replacement.

Additional difficulties appear when alternatives are described with verbal attributes. The operations of averaging, weighing, mixing and similar data transformations are mathematically incorrect and unacceptable for qualitative variables. Therefore, a group of several corteges with verbal attributes cannot be simply transformed into a single cortege. Thus, we need new ideas to aggregate such corteges and operate with them.

There is another approach to representing multi-attribute objects described with quantitative and/or qualitative attributes. Define the combined attribute scale or the hyperscale that is a set $X = X_1 \cup \ldots \cup X_m$ consisting of m attribute (criteria) scales $X_i = \{x_i^{ei}\}$. And represent an object A_q as the following set of repeated attributes:

$$\boldsymbol{A}_q = \{k_{\boldsymbol{A}_q}(x_1^1) \circ x_1^1, \ldots, k_{\boldsymbol{A}_q}(x_1^{g1}) \circ x_1^{g1}, \ldots, k_{\boldsymbol{A}_q}(x_m^1) \circ x_m^1, \ldots, k_{\boldsymbol{A}_q}(x_m^{gm}) \circ x_m^{gm}\}.$$

Here $k_{\boldsymbol{A}_q}(x_i^{ei})$ is the number of attribute x_i^{ei}, which is equal to the number of ways for evaluating the object A_q with the attribute $x_i^{ei} \in X_i$; the sign $\circ$ denotes that there are $k_{\boldsymbol{A}_q}(x_i^{ei})$ copies of attribute x_i^{ei} included in the description of object A_q.

Thus, the object A_q is represented as a set of many repeated elements (criteria estimates) x_i^{ei} or as a multiset $\boldsymbol{A}_q$ over the domain X that is defined by a multiplicity function $k_A : G \to \mathbf{Z}_+ = \{0, 1, 2, 3, \ldots\}$. A multiset $\boldsymbol{A}_q$ is said to be finite when all $k_{\boldsymbol{A}_q}(x_i^{ei})$ are finite. A multiset $\boldsymbol{A}_q$ becomes a crisp set A_q when $k_{A_q}(x_i^{ei}) = \chi_{A_q}(x_i^{ei})$, where $\chi_{A_q}(x_i^{ei}) = 1$ for $x_i^{ei} \in A_q$, and $\chi_{A_q}(x_i^{ei}) = 0$ for $x_i^{ei} \notin A_q$. Multisets $\boldsymbol{A}$ and $\boldsymbol{B}$ are said to be equal ($\boldsymbol{A} = \boldsymbol{B}$), if $k_{\boldsymbol{A}}(x_i^{ei}) = k_{\boldsymbol{B}}(x_i^{ei})$. A multiset $\boldsymbol{B}$ is said to be included in a multiset $\boldsymbol{A}$ ($\boldsymbol{B} \subseteq \boldsymbol{A}$), if $k_{\boldsymbol{B}}(x_i^{ei}) \leq k_{\boldsymbol{A}}(x_i^{ei})$, $\forall\, x_i^{ei} \in X$. In order to compare, arrange or classify multi-attribute objects $A_1, \ldots, A_p$, they are considered as points in the multiset metric space with the different types of distances.[6–8]

In MCDA, it is important to incorporate DM preferences. The person expresses his/her preferences when he/she describes properties and characteristics of the analyzed problem, compares decision alternatives, estimates the quality of the choice. Preferences may be represented as decision rules of mathematical, logical and/or verbal nature and explained in any language. While solving the problem, a person may behave inconsistently, make errors and contradictions. In the case of individual choice, the consistency of subjective preferences is postulated. Hence, in order to discover and correct possible inconsistent and contradictory judgments of a single DM, special procedures are to be included in MCDA methods.

A collective choice by several independent actors is more complicated and principally different due to a variety and inconsistency of many subjective preferences. Each DM may have his/her own personal goals, interests, valuations and information sources. As a result, individual subjective judgments of several persons may be similar, concordant or discordant. Usually, in MCDA techniques, one tries to avoid possible inconsistencies and contradictions between judgments of several persons. In this case, the number of opposite opinions is replaced with a common preference that mostly agrees with all individual points of view. Nevertheless, individual preferences may not always be coordinated.

Let us consider some MCDA methods to sort multi-attribute objects.

Object arrangement by pairwise comparisons is a widely popular technique. The order is complete if all objects are comparable, and DM preferences are transitive. The order is partial if some objects are incomparable. In the case of multiple criteria and/or several persons, the final arrangement of objects by comparing many vectors is difficult, for example, in MAUT and TOPSIS methods.[4,5] In AHP,[9] priorities of alternatives and criteria are derived by hierarchical pairwise comparisons with respect to their contribution to the problem solution. Objects can be arranged also by their ranks, which are calculated or evaluated by DM. Ordering multi-attribute objects with TOMASO tool[10] are produced by an aggregation of families of discriminant functions.

In the ELECTRE family of methods,[11,12] objects are assessed using many numerical criteria with different weights specified by the DM. Alternatives are compared by the outranking relation and ordered or assigned to given classes according to their boundaries. Ranks of alternatives and boundaries of classes are determined by the special indexes of concordance and discordance. These indexes are calculated for pairwise compared alternatives without any strict verification.

An interactive classification has been proposed in Ref. 13. This method is based on DM preferences represented by a linear utility function as a weighted sum of many scalar criteria. A linear aggregation model to flexibly rank order or sort the multidimensional alternatives using the idea of "outranking" as many of the competing alternatives has been developed in Ref. 14. In this model the criterion weights can be varied within reasonable ranges.

In the rough sets methodology for sorting multicriteria alternatives, DM preferences are described with decision rules, which suggest an assignment of the alternative to a given class with different approximations.[15,16] The rough set technique operates with a big collection of sorting decision rules that is difficult for DM analysis and demands specific learning on training samples.

In case of many criteria and/or actors, ranking and sorting objects are more complicated due to errors and inconsistencies of DM. While methods using convolution of many criteria are applied to MCDA problems of large dimension, an explanation of the obtained results becomes difficult due to an impossibility to recover aggregated input data. Convolution of criteria and transformation of verbal estimates into numerical scores are not clear for the DM and do not allow to explain the obtained results by using initial information.

In verbal decision analysis, alternatives and decision classes are described using qualitative criteria possessing verbal scales.[3,17] Neither numerical coefficients of the criterion importance, nor values of utility functions are estimated and applied. Verbal grades on criteria scales are not converted into any numerical data. Thus, using only qualitative measurements, relations of the superiority and equivalence of objects are given on a set $X_1 \times \cdots \times X_m$ of tuples. DM preferences are checked on consistency, and the revealed inconsistencies are excluded. Based on these relations, DM constructs a classification of alternatives, finds a partial ordering or selects the best option. In the ZAPROS (the abbreviation of Russian words: Closed Procedures nearby Reference Situation) family of methods, the joint ordered scale is built and used for objects' arrangement. In ORCLASS (ORdinal CLASSification) and CYKLE (the abbreviation of Russian words: Chain Interactive CLassification) methods, a complete and consistent ordinal classification of multicriteria alternatives is built. Boundaries of decision classes are determined by subsets of corteges with verbal components.

Group verbal decision analysis is a new approach in MCDA, which enlarges verbal decision analysis methodology to group decisions.[1,2,7,8,18,19] Methods of group verbal decision analysis take into account the preferences of several DMs and do not require finding a compromise between the inconsistent and discordant judgments. In these methods, multi-attribute objects are represented as multisets. The ARAMIS (Aggregation and Ranking Alternatives nearby the Multi-attribute Ideal Situations) method allows ordering multi-attribute objects by a group without a pre-construction of individual ranking objects. The objects are arranged with respect to closeness to the best or the worst object in the multiset metric space introduced by the author in 1994.[6] The MASKA (abbreviation of the Russian words Multi-Attribute Consistent Classification of Alternatives) method is used to sort multi-attribute objects by a group. This method allows the DM to construct a group decision rule that aggregates inconsistent individual expert rules for sorting objects.

The methods mentioned above work well enough in the attribute space of small dimension. When the objects are described by a large number of attributes, the DM experiences some difficulties in comparing and classifying such multi-attribute objects because many of them are incomparable, in general.

3. Technique of MCDA in Reduced Attribute Space

Let us consider the main ideas of a new multi-stage technique PAKS (the abbreviation of Russian words Consequent Aggregation of Classified Situations) to solve a problem of multicriteria choice. This technique combines procedures of reducing the dimension of the attribute space and different methods of verbal decision analysis.

At the first stage, the composite criteria and the complex indicator of the top level are formed with the original interactive procedure HISCRA (HIerarchical Structuring CRiteria and Attributes) for reducing the dimension of the attribute space.[20] A composite criterion (complex indicator) is an integrated index, which characterizes the object property selected by the DM and aggregated initial characteristics. Each gradation on a scale of composite criterion is a combination of grades of initial attributes that has a concrete context for the DM. The DM defines a block structure for a hierarchical list of criteria, the number and context of criteria, their scales at every level of hierarchy.

At the second stage, gradations of the formed composite criteria are composed progressively, step by step, using various methods of verbal decision analysis.

A construction of composite criterion scale is considered as ordinal classification, which is obtained with different methods. In this procedure, the classified alternatives are combinations of attributes, and the decision classes are verbal gradations of a composite criterion. Thus, each object with concrete values of attributes is assigned to any class corresponding to the grade of the composite criterion.[21,22]

At the third stage, all objects are sorted into the generalized classes by the ARAMIS method for group ordering multi-attribute objects presented as multisets in the reduced space of new composite criteria.[8] The suggested technique provides effective tools for solution of multiple criteria problems, allows DM to analyze and explain the final results, and essentially diminish the time for solving the problem.

The offered approach to attributes' aggregation and composite criterion construction is based, in general, on DM preferences. First of all, a set of initial characteristics for the considered collection of objects is to be formed. These characteristics are determined by the problem specificity and may be given beforehand or generated in the course of problem analysis. Further, based on DM experience and intuition, a hierarchical system of criteria is constructed. DM establishes, which initial attributes are to be considered as independent final criteria and which will be combined into composite criteria. DM can combine the criteria consistently in groups, for example, based on a sense of "similarity".

DM also defines the semantic content of the criteria and grades of the scale. Grades of the criteria scales, on the one hand, should reflect the aggregated properties of the objects, on the other hand, be clear to DM in the final arrangement or classification of objects. We recommend to build scales of criteria with a small number (3–5) of verbal grades. In order to reduce the influence of the peculiarities of different methods, which are used to form scales of the composite criteria, it is proposed to apply several different methods and/or their combinations at various stages of the procedure.

Thus, the DM can create various ways to build the set of composite criteria and the integrated indicator, whose grades are used to solve the applied problem. This allows the DM to analyze and compare the results obtained for the different options of criteria aggregation, assess the quality of the final solution of the original problem. Such a procedure is similar to Keeney's ideology of value-focused thinking[23] and Zadeh's idea of information granulation,[24] but does not use any value and/or membership functions.

We now discuss in more detail the problem of reducing the dimension of the attribute space. This problem is formulated as follows:

$$X_1 \times \cdots \times X_m \to Y_1 \times \cdots \times Y_n, \quad n < m,$$

where $X_1, \ldots, X_m$ are the sets of initial attributes, $Y_1, \ldots, Y_n$ are the sets of new attributes, m and n are dimensions of the initial and new attribute spaces. In our case, the attribute sets $X_i = \{x_i^1, \ldots, x_i^{gi}\}$, $i = 1, \ldots, m$, and $Y_j = \{y_j^1, \ldots, y_j^{hj}\}$, $j = 1, \ldots, n$ are supposed to be ordered.

Present the above problem as a problem of ordinal classification of multicriteria alternatives. Different combinations of initial attributes (or corteges of criteria grades) in the space $X_1 \times \cdots \times X_m$ are considered as alternatives aggregated into smaller sets of classes (categories) $Y_1, \dots, Y_n$ with ordered scales. The verbal grades of every new attribute Y_j have a concrete context for the DM. Attributes are aggregated consequently, step by step. The obtained groups of criteria may be combined, in turn, into new groups at the following level of hierarchy, and so on. By combining attributes into a small number of composite criteria, the DM can form a hierarchical system of criteria up to a single top criterion (complex indicator), whose grades correspond to the decision classes.

The suggested procedure of building the composite criterion scale has a block structure and consists of several unified blocks of classification executed step by step. The DM selects these blocks depending on the problem specifics. Each classification block of ith hierarchical level includes any attribute set and a single composite criterion. Corteges of scale grades of the initial attributes represent the classified objects. Grades on the scale of the composite criterion provide the decision classes of the ith level.

In a classification block of the next hierarchical level, the composite criteria of the ith level are considered to be new attributes. Corteges of their scale grades represent new classified objects in the reduced attribute space, whereas the decision classes of the $(i+1)$th level will now be the scale grades of the new composite criterion. The procedure is repeated up to a single composite criterion (complex indicator) of the top hierarchical level, whose scale provides the required ordered classes $D_1, \dots, D_l$. Thereby a correspondence between classes $D_1, \dots, D_l$ and a collection of corteges $(x_1^{e1}, \dots, x_m^{em})$ of initial attributes in the space $X_1 \times \cdots \times X_m$ is established. The found boundaries of classes allow to sort easily the real multiple criteria objects (alternatives) $A_1, \dots, A_p$ estimated upon many criteria $K_1, \dots, K_m$.

Consider a small illustrative example. Suppose that the DM needs to form a scale for a composite criterion D from the grades of three initial attributes A, B, C. Let the scales of the composite criterion and each attribute have three verbal grades as follows: $D = \{d^1, d^2, d^3\}, A = \{a^1, a^2, a^3\}, B = \{b^1, b^2, b^3\}$, $C = \{c^1, c^2, c^3\}$, where, for instance, x^1 is the 'excellent' grade, x^2 is the 'middle' grade, x^3 is the 'poor' grade.

To build scales of composite criteria, DM can use different procedures. The simplest way for forming a scale of the composite criterion is the tuple (cortege) stratification technique that is based on cutting the multi-attribute space with parallel hyperplanes. Each layer (stratum) consists of combinations of the unified initial grades with fixed sum of numbers and represents any generalized grade on the scale of the composite criterion. The DM determines a number of layers (or scale grades). The maximal number of layers is equal to $s = 1 - m + \Sigma_i g_i$. The number of classes is equal to $r \leq s$. By the tuple stratification technique the DM can combine the initial grades into the generalized grades of the composite criterion, for example, as follows: the best initial grades form the best generalized grade, middle initial

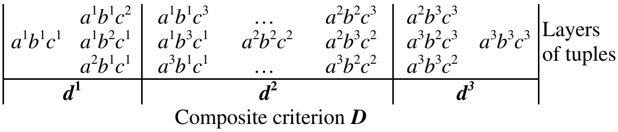

Fig. 1. Scale of the composite criterion obtained by the tuple stratification technique.

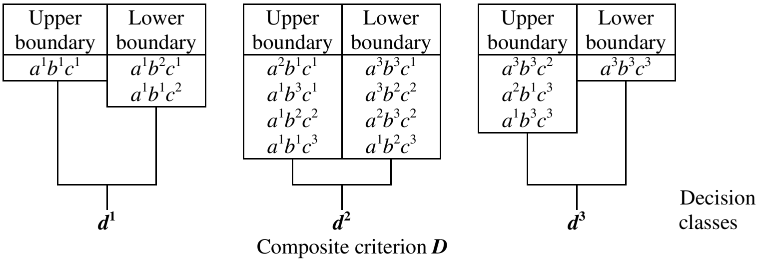

Fig. 2. Scale of the composite criterion obtained by the ORCLASS method.

grades form the middle generalized grade, and the worst initial grades form the worst generalized grade (Fig. 1).

A more complicated composition of the composite criterion scale uses the methods of verbal decision analysis.[3,17] In these cases, all the possible combinations of initial grades in the attributes space are considered as multi-attribute alternatives, whose number is equal to $t = \Pi_i g_i$. The ORCLASS and CYCLE methods provide a complete and consistent classification of all the corteges of initial grades. The decision classes are determined by their upper and lower boundaries, which form an ordinal scale for the composite criterion (Fig. 2).

Various ways for forming scales for the composite criteria may be used at different stages of the aggregation procedure. For example, the tuple stratification technique may be used for generating some composite criteria, and multicriteria ordinal classification for generating other criteria. The considered problem of multiple criteria choice is solved at the last stage of the procedure in the new reduced attribute space.

Thus, the PAKS technique for building the hierarchy of composite criteria by reducing the dimension of the attribute space includes the following steps (Fig. 3).

Step 1. Select the type T of the multiple criteria problem as follows: T_1 — to find the best alternative(s); T_2 — to order alternatives; T_3 — to assign alternatives to ordered classes.

Step 2. Form a set of the real alternatives (objects, variants) $A_1, \dots, A_p$, $p \geq 2$.

Step 3. Form a set of the initial attributes (criteria) $K_1, \dots, K_m$, $m \geq 2$, which form the lower level of the hierarchical system of attributes. These attributes can either be specified in advance (for example, technical characteristics of the object) or generated in the course of problem analysis.

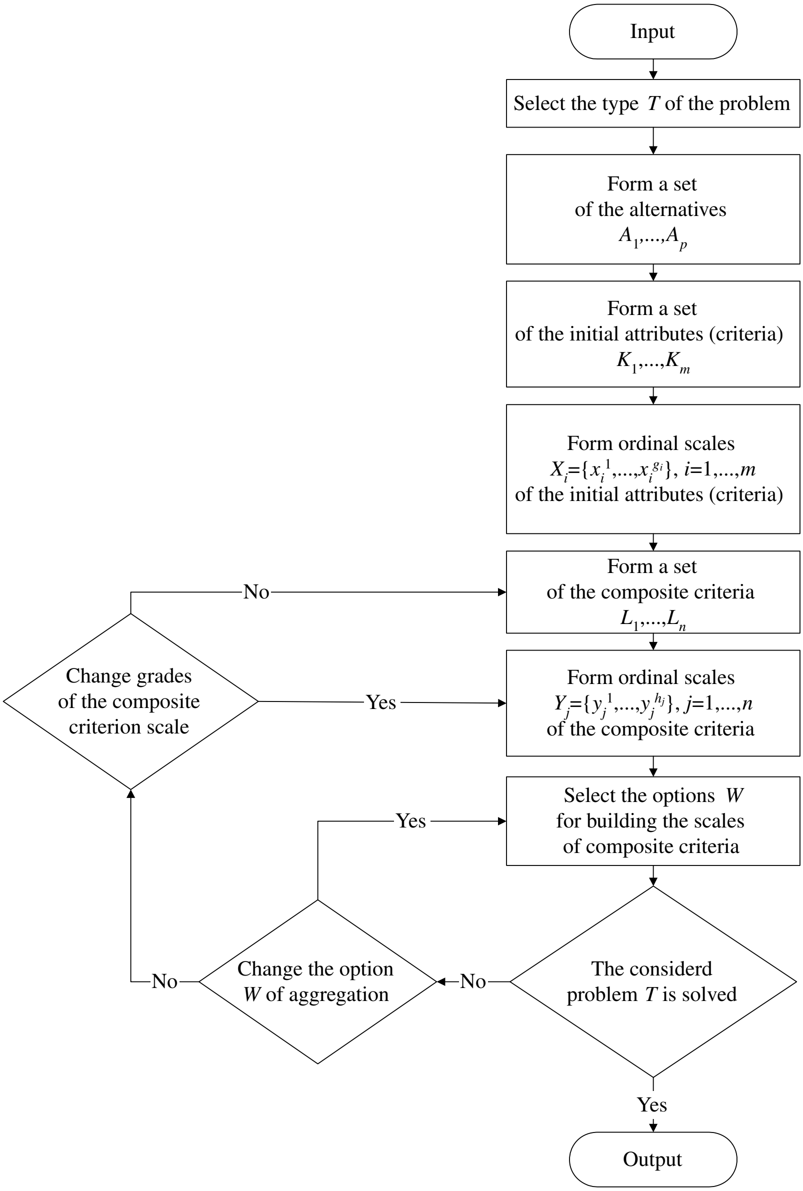

Fig. 3. Block diagram of the PAKS technique.

Step 4. Form ordinal scales $X_i = \{x_i^1, \ldots, x_i^{gi}\}$, $i = 1, \ldots, m$ of the initial attributes. A scale of an attribute may be numerical (point-wise, interval) or verbal, whose grades coincide with those really used in practice or constructed specially. The set of tuples formed the Cartesian product $X_1 \times \cdots \times X_m$ of the initial attributes, are considered as the set of all possible alternatives.

Step 5. Form a set of the composite criteria (complex indicators) $L_1, \ldots, L_n$, $n < m$, which define the aggregated properties of objects selected by DM.

Step 6. Form ordinal scales $Y_j = \{y_j^1, \ldots, y_j^{hj}\}$, $j = 1, \ldots, n$ of the composite criteria. Each grade of the composite criterion scale is a combination of initial attributes.

Step 7. Select the options W for building the scales of composite criteria at all hierarchical levels (including the upper level): W_1 — a stratification of tuples, W_2 — an arrangement of tuples, W_3 — a classification of tuples.

Step 8. Build scales for all composite criteria using several different methods and/or combinations of methods for aggregation of attributes.

Step 9. Solve a problem T. If DM accepts the obtained result, then stop. Otherwise go to step 10.

Step 10. If the result obtained at step 8 is unsatisfactory, then it is possible either to change the option for building the scale for the composite criterion (go to step 7), or change grades of the composite criterion scale (go to step 6), or form a new set of composite criteria (go to step 5).

This methodological approach to solving specific multiple criteria problems allows the DM to determine the preferred list of composite criteria and select the method or a combination of methods for building scales for criteria at different stages of the aggregation procedure. The proposed multi-stage technique for reducing the dimension of attribute space has a certain universality, because, in the general case, one can operate with symbolic (qualitative) and numerical (quantitative) data. An important feature of PAKS is using procedures, both independently and in conjunction with other methods of ranking and/or classification of multicriteria alternatives.

4. Multicriteria Evaluation of R&D Project Efficiency

The Russian Foundation for Basic Research (RFBR) is the federal agency that organizes and subsidizes basic research, evaluates the efficiency of the research projects and examines the practical usefulness of the results. One of the important RFBR activities is support of the goal-oriented R&D projects performed for federal Agencies and Departments of Russia.[22,25] The RFBR possesses an extensive experience in organizing, conducting and estimating basic research. In RFBR, there is a special peer review system for grant applications and completed projects. Experts are well-known specialists working in research institutes, universities, and industrial organizations.

Every project is estimated independently by several experts without any coordination of their judgments. In order to assess the content of application or research results, each expert uses specific qualitative criteria with detailed verbal scales. In addition, an expert gives recommendations whether to support a project (at the competition stage) or continue the project (at the intermediate stage). At the final stage, an expert estimates the scientific and practical value of the obtained results. Expert opinions, of course, may be close to each other or different. Based on expert recommendations, the Expert Board of RFBR decides to approve or reject the new projects, continue supporting the implemented projects, and evaluates the efficiency of the completed projects.

Numerous technologies for assessment of programs and projects of various kinds are known and widely used in practice. We mention such tools as 'Peer review', 'Cost-Effectiveness', 'Programming-Planning-Budgeting', 'Balanced Score Card'. Most of the methodologies applied for expert estimation of different projects use a quantitative approach that is based on a numeric measurement of object characteristics. However, such quantitative approaches are not suitable for the RFBR expertise, where several experts evaluate R&D projects by many qualitative criteria with verbal scales.

For instance, two or three experts estimate each completed goal-oriented R&D project by the following eight criteria: K_1 "Degree to which the problem has been solved", K_2 "Scientific level of results", K_3 "Protection of results", K_4 "Prospects of applying the results", K_5 "Results correspondence to the project goal", K_6 "Goal achievement", K_7 "Difficulties in the project performance", K_8 "Collaboration with users". Each criterion has 2- or 3-point scale $X_i = \{x_i^1, x_i^2, x_i^3\}$, $i = 1, \dots, 8$ of verbal grades. For example, "Degree to which the problem has been solved" is estimated as x_1^1 — 'the problem is solved completely', x_1^2 — 'the problem is solved partially', x_1^3 — 'the problem is not solved'. The criterion "Achievement of the project goal" has the scale with grades x_6^1 — 'really', x_6^2 — 'nonreally'. For simplicity of notation, verbal grades can be below replaced by symbols (not numbers) as follows: 0 is the best grade x_i^1, 1 is the middle (or the worst) grade x_i^2, 2 is the worst grade x_i^3.

In order to evaluate the applicability of the obtained results, a notion of "Project efficiency" has been formalized. Constructing the complex indicator of project efficiency was examined as the multiple criteria classification problem in the attribute space of the reduced dimension. The classified alternatives were combinations of multicriteria estimations in the initial attribute space $X_1 \times \cdots \times X_8$. The final decision classes were the gradations on the scale $Z = \{z^1, z^2, z^3, z^4, z^5\}$ of the top-level indicator D "Project efficiency". These gradations correspond to grades of project efficiency as follows: z^1 — 'superior', z^2 — 'high', z^3 — 'middle', z^4 — 'low', z^5 — 'unsatisfactory'.

Suppose that the DM aggregated the initial criteria K_1, K_2, K_3 into a composite criterion AK_1 "Project results", the initial criteria K_5, K_6, K_7 into a composite criterion AK_2 "Project realization", and the initial criteria K_4, K_8 into a composite criterion AK_3 "Application of project results". The composite criteria AK_1, AK_2, AK_3 have scales $Y_j = \{y_j^1, y_j^2, y_j^3\}$, $j = 1, 2, 3$. Here the verbal grades (y_j^1 — 'high', y_j^2 — 'middle', y_j^3 — 'low') designate the decision classes of the first hierarchical level and may be replaced, as above, by the symbol 0, 1, 2. The name of the class depends on the context of the corresponding criterion.

The composite criteria AK_1, AK_2, AK_3 are integrated into the complex indicator D "Project efficiency" of the top hierarchical level. DM can form composite criteria

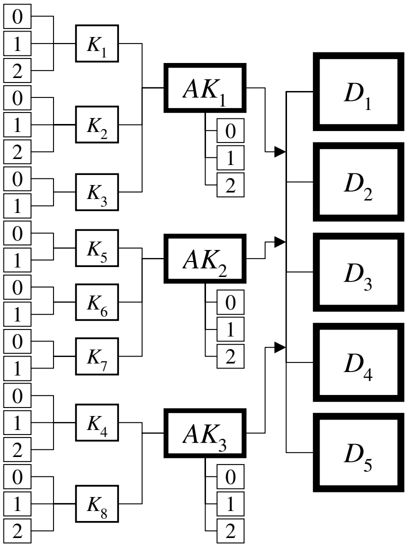

Fig. 4. Hierarchical frame of building composite criteria and composing rating scales.

and the integrated indicator of project efficiency in different ways, compare the constructed indexes and the obtained results. The hierarchical frame of building composite criteria and forming their scales is presented in Fig. 4.

While constructing the scales of composite criteria AK_1, AK_2, AK_3, the DM used, for instance, the ORCLASS method. In accordance with DM preferences, for the composite criteria AK_1 the class 'high project results' (grade y_1^1 or 0) includes the following combinations of grades: (000), (001), (010), (100); the class 'average project results' (grade y_1^2 or 1) includes the combinations of grades (011), (021), (101), (111), (201), (110), (200), (020), (210), (120); the class 'poor project results' (grade y_1^3 or 2) includes the combinations of grades (121), (211), (221), (220). For the criteria AK_2 the class 'high project realization' (grade y_2^1 or 0) consists of the best grades (000); the class 'average project realization' (grade y_2^2 or 1) consists of the middle grades (001), (011), (101), (100), (010), (110); the class 'poor project realization' (grade y_2^3 or 2) consists of the worst grades (111). For the criteria AK_3 the class 'results will be used to full extent' (grade y_3^1 or 0) includes the combination of grades (00); the class 'results will be used partially' (grade y_3^2 or 1) includes combinations of grades (01), (10), (02), (11), (20); the class 'results will be used poorly' (grade y_3^3 or 2) includes the combinations of grades (12), (21), (22). The scales of the composite criteria AK_1, AK_2, AK_3 are shown in Fig. 5. The commas between components of tuples are omitted for simplicity of notation.

Consider now the collections of all corteges $(y_1^{h1}, y_2^{h2}, y_3^{h3})$ in the space $Y_1 \times Y_2 \times Y_3$, that consists of grades on the scales of composite criteria AK_1, AK_2, AK_3, as the new classified objects of the next hierarchical level in the criteria list. The decision classes $D_1, \ldots, D_5$ are the grades on the scale $Z = \{z^1, z^2, z^3, z^4, z^5\}$ of the

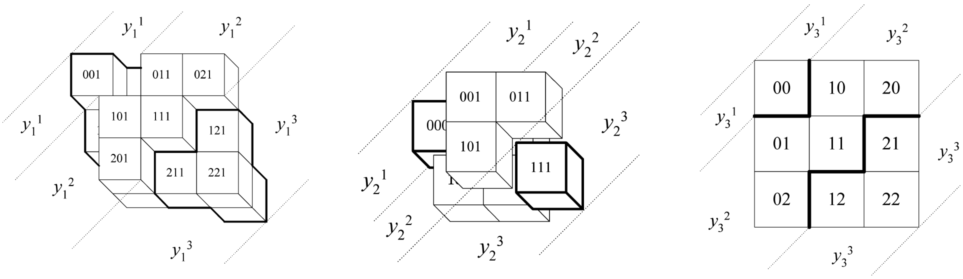

Fig. 5. Scales of the composite criteria AK_1, AK_2, AK_3.

complex indicator (composite criterion) D "Project efficiency" at the top level of the hierarchy. Combining the attributes of composite criteria AK_1, AK_2, AK_3 with the tuple stratification technique, one may obtain the following results. The class D_1 'Superior efficiency' (grade z^1) consists of the best estimates (000). The class D_2 'High efficiency' (grade z^2) consists of the estimates (100), (010), (001), (002), (101), (011), (200), (110), (020). The class D_3 'Average efficiency' (grade z^3) consists of the estimates (102), (012), (201), (111), (021), (210), (120). The class D_4 'Low efficiency' (grade z^4) consists of the estimates (202), (112), (022), (211), (121), (220), (212), (122), (221). The class D_5 'Unsatisfactory efficiency' (grade z^4) consists of the worst estimates (222). The scale of the composite criterion Z is shown in Fig. 6. Thus, the real objects estimated by initial criteria are assigned directly to the generated decision classes. Note that this procedure essentially requires less efforts than other methods of multicriteria ordinal classification.

The grades of the integrated indicator D "Project efficiency" were formed with the following different methods of verbal decision analysis. M_1 — the ORCLASS method was used at all levels of the criteria hierarchy (OC). M_2 — the stratification of tuples was used at all levels of the criteria hierarchy (ST). M_3 — the stratification of tuples was used at the lower level of the criteria hierarchy, and the ORCLASS method was used at the upper level of the criteria hierarchy (ST+OC). M_4 — the ORCLASS method was used at the lower level of the criteria hierarchy, and the

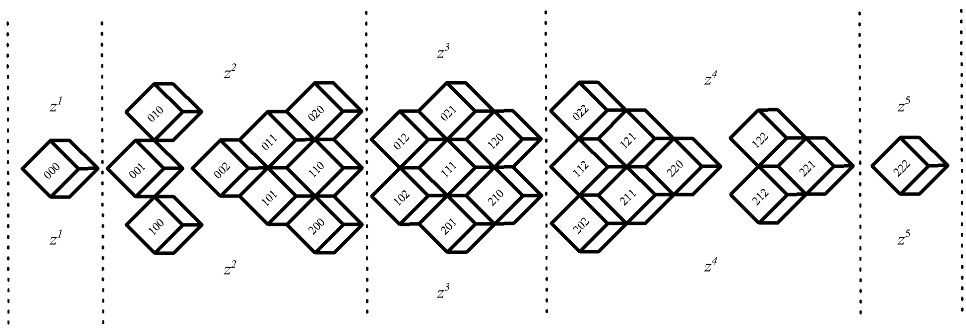

Fig. 6. Scale of the composite criterion (complex indicator) Z.

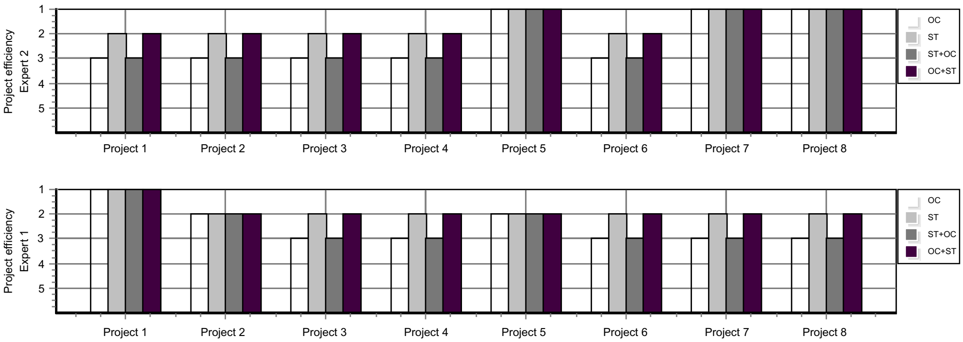

Fig. 7. Integrated indicators of project efficiency.

stratification of tuples was used at the upper level of the criteria hierarchy (OC+ST).

The model database of research results consists of RFBR expert assessments of the goal-oriented R&D projects. These projects were completed in 2007 in Mathematics, Mechanics and Computer Science (total 48 projects), Chemistry (total 54 projects), Information and telecommunication resources (total 21 projects). Two experts evaluated each project by eight criteria K_1–K_8. Examples of the complex indicators of project efficiency are shown in Fig. 7. The classes of project efficiency coincide in 74% and 48% cases (for the area 01); in 72% and 24% cases (for the area 03); in 76% and 62% cases (for the area 07). The first number is related to the projects estimated by the first expert, and the second number to the projects estimated by the second expert.

In the considered case, we studied two different ways to aggregate the integrated indicator D "Project efficiency" of the top level and construct the hierarchy of criteria. In option A, the initial indicators K_1–K_3, K_5–K_7, and K_4, K_8 were combined into three intermediate criteria. In option B, the initial indicators K_1–K_4 and K_1–K_4 were combined into two intermediate criteria. So, for instance, in the area 3, 6 and 16 projects possessed superior efficiency, 40 and 75 projects had high efficiency, 59 and 13 projects had average efficiency, 1 and 2 projects had low efficiency, 2 and 2 projects had unsatisfactory efficiency. The first number of projects is related to the option A and the second to the option B. In general, the values of project efficiency coincided in 41 cases out of 108. In other cases, the integrated indicators differed by no more than one gradation. These data confirm the high stability of the suggested approach to build the hierarchy of criteria and construct the integrated indicators of project efficiency.

To find the best projects, we applied the ARAMIS method for ordering multi-attribute objects. Let the methods M_1, M_2, M_3, M_4, which have been used by experts to assess the project efficiency, be new attributes characterizing the projects. Every attribute M_j takes values m_j^1, m_j^2, m_j^3, m_j^4, m_j^5, which correspond to decision classes D_1, D_2, D_3, D_4, D_5 of project efficiency. Represent now each project A_i as the following multiset

$$\boldsymbol{A}_i = \{k_{\boldsymbol{A}_i}(m_1^1) \circ m_1^1, \ldots, k_{\boldsymbol{A}_i}(m_1^5) \circ m_1^5; \ldots; k_{\boldsymbol{A}_i}(m_4^1) \circ m_4^1, \ldots, k_{\boldsymbol{A}_i}(m_4^5) \circ m_4^5\}$$

over the set of methods $M = M_1 \cup M_2 \cup M_3 \cup M_4$. Here multiplicity $k_{\boldsymbol{A}_i}(m_j^{hj})$ of each attribute value in the multiset $\boldsymbol{A}_i$ indicates how many times the method m_j^{hj}, $h_j = 1, \ldots, 5$, $j = 1, \ldots, 4$ was used by all experts during a formation of the appropriate class of efficiency. The sign $\circ$ denotes that there are $k_{\boldsymbol{A}_i}(m_j^{hj})$ copies of attribute m_j^{hj} within the description of project A_i.

For instance, the projects A_1 and A_2 shown in Fig. 7 are represented as the following multisets:

$$\begin{aligned}
\boldsymbol{A}_1 = \{&1 \circ m_1^1, 0 \circ m_1^2, 1 \circ m_1^3, 0 \circ m_1^4, 0 \circ m_1^5; 1 \circ m_2^1, 1 \circ m_2^2, 0 \circ m_2^3, 0 \circ m_2^4, 0 \circ m_2^5;\\
&1 \circ m_3^1, 0 \circ m_3^2, 1 \circ m_3^3, 0 \circ m_1^4, 0 \circ m_1^5; 1 \circ m_4^1, 1 \circ m_4^2, 0 \circ m_4^3, 0 \circ m_4^4, 0 \circ m_4^5\},\\
\boldsymbol{A}_2 = \{&0 \circ m_1^1, 1 \circ m_1^2, 1 \circ m_1^3, 0 \circ m_1^4, 0 \circ m_1^5; 0 \circ m_2^1, 2 \circ m_2^2, 0 \circ m_2^3, 0 \circ m_2^4, 0 \circ m_2^5;\\
&0 \circ m_3^1, 1 \circ m_3^2, 1 \circ m_3^3, 0 \circ m_1^4, 0 \circ m_1^5; 0 \circ m_4^1, 2 \circ m_4^2, 0 \circ m_4^3, 0 \circ m_4^4, 0 \circ m_4^5\}.
\end{aligned}$$

The best (ideal) and the worst (anti-ideal) projects (may be hypothetical) have the highest and lowest estimates by all attributes. The best project A_+ and the worst project A_- are represented as multisets

$$\begin{aligned}
\boldsymbol{A}_+ &= \{2 \circ m_1^1, 0, \ldots, 0; 2 \circ m_2^1, 0, \ldots, 0; 2 \circ m_3^1, 0, \ldots, 0; 2 \circ m_4^1, 0, \ldots, 0\},\\
\boldsymbol{A}_- &= \{0, \ldots, 0, 2 \circ m_1^5; 0, \ldots, 0, 2 \circ m_2^5; 0, \ldots, 0, 2 \circ m_3^5; 0, \ldots, 0, 2 \circ m_4^5\}.
\end{aligned}$$

Considering multi-attribute objects $A_1, \ldots, A_p$ as points in a multiset metric space, one can compare and arrange objects with respect to closeness to the best or worst object in this space. Thus, for instance, in the final ranking of projects in Mathematics, Mechanics and Computer Science, 23 projects had superior efficiency, 1 project had efficiency between superior and high, 24 projects had high efficiency.[25]

5. Conclusion

In this paper, we have suggested a new approach to sorting alternatives estimated by many criteria with numerical, symbolic, and verbal scales. By using the PAKS technique in practice, the DM has the possibility to generate different lists of criteria, determine the most preferable composite criteria, select the method or combination of methods for building the hierarchy of criteria, and compare the obtained results for different sets of criteria. The developed procedure of hierarchical aggregation provides tools to systematically investigate the available information, find the best solution, analyze and explain the final decision. Note that the PAKS technique for reducing the dimension of the attribute space can be applied in conjunction with different methods of information processing and decision making, including the ones mentioned in this article.

We have also proposed a new transparent approach to building the complex indicator (index) of activity, which integrates many initial attributes. This approach was tested on goal-oriented R&D projects supported by the Russian Foundation for Basic Research. We ranked the projects and found the most effective projects. The proposed approach allows to discover, present and utilize the available information, to analyze the obtained results and their peculiarities, essentially diminishing the time for solving the problem. The multi-stage, multi-method technique PAKS for MCDA demonstrated good qualities, especially in cases of inconsistent multiple criteria estimates of objects and contradictory preferences of decision makers. We underline that available information is only used in the original form without any transformation of verbal data into numbers.

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