Multicriteria Problems with Objective Models

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Introduction

The multicriteria mathematical programming problems to be considered at the conference, are quite specific differing from both operations research problems and many mul-tiattribute decision problems.

In multicriteria mathematical programming problems, like in many operations research problems, there is a reliable (objective) model of the object under study, i.e. a set of perfectly verified relationships between the basic object variables. However in contrast to operations research, there is a variety of requirements to the quality of solution, i.e. multiplecriteria.

The latter constitutes a specific characteristic of the widespread real-life problems. The choice of the best decision alternative places demand for a tradeoff between the estimates against different criteria. The problem conditions lack information permitting a tradeoff. Hence, it cannot be found through objective calculations.

The analysis of many real-life problems, the operations researchers had dealt with, has naturally produced a class of multicriteria problems lacking information which makes it possible to find the best decision.

Since a decision must, somehow or other, be made, the shortage of information required for the best alternative choice, should be eliminated. This can be done only by people on the basis of experience and intuition.

The evolvement of preferences and human policy as an inseparable part of the problem drastically changes both its essence and solution techniques. There arises a plethora of questions characteristic of all decision problems:

1. How to help man validate the rationality of his decision?
2. How to elicit information from man in the process of problem solution?
3. How to verify the consistency of information elicited from man?
4. How to help man analyze the opportunities for a tradeoff between the criteria, determined by the objective model of the considered problem?

The four questions, we believe, axe fundamental for constructing man-machine solution procedures of multicriteria problems with objective models.

Hence we infer from here that transition from single-

Now we turn to the search for answers to the above questions using a multicriteria linear programming problem (MLPP) as an example:

Find vector $x = (x_1,\ldots,x_n)$ belonging to domain

$$D = \{Ax = b;\ x_i \geq 0,\ i = 1,\ldots,\ n\}$$

where A is p x n-matrix; b is p-vector maximizing (or minimizing) the set of objective functions

$$c_k(x) = \sum_{i=1}^{n} c_{ik} x_i; \qquad k=1,\ldots,N$$

for the most preferable ratio between their values in decision point. This requirement means: in a variety of X effective (Pareto-optimal) decisions one should seek for X^* decisions corresponding to the extremum of a priori unknown decision maker's utility function. Analysis shows that there are three groups of methods developed by different authors for solving MLPP. According to one of them, at the analysis phase decision maker compares changes in the estimates of a pair of criteria and/or assigns a satisfactory value against one criterion. This idea was first advanced in STEM procedure (Benayon et al., 1971). According to the second idea, decision maker specifies direction in the criterion space along which his implicit utility function increases (analogy of gradient method). The most familiar procedure of this type is the one of Dyer-Gioffrion (Dyer, 1976). The third version of man-machine procedure construction boils down to gradual localization of e-vicinity of optimal point, and is related with truncation of feasible decision domain.

Validation of decision rationality

Though solution of multicriteria problems with objective models depends on the decision maker preferences, this does not imply that he "makes whatever he likes". An individual must be rational in business decisions so that to be able to convince others, explain to them the motives of his choice, the logics of his subjective model. Any decision maker preferences should, therefore, be within the frameworks of some rational system. Very often his policy, his subjective model is, in fact, manifestation of the policy of a group of persons surrounding him. This does not make the model more objective, rather it becomes as if more stable — it remains the same for any decision maker from some group possessing a common preference, a common "world outlook". Often this unity is largely determined by the status of the organization, the given group of managers belongs to, its environments.

This forces the decision maker to explain the derived decision. Hence, on arriving to a decision, he has first to trace the logics of his successive decisions for himself and only then explain this logics to others.

Different methods of MLPP solution provide the decision maker with different opportunities for explaining the choice. Thus, methods involving assignment of the so-called "ideal" decisions (procedures by M. Zeleny, 1976; A. Wierzbicki, 1980) provide a good opportunity for explanations in the form of an "ideal" points trajectory. Transition from one "ideal" point to the other can be explained by a real decision obtained on the margin of a feasible domain. As acceptable for explanation are the methods of alternate assignment of satisfactory criteria values (Spronk method, Nijkamp and Spronk, 1980; STEM, Benayon et al., 1971). Explanation can be in the form of a set of curves of tradeoffs between pairs of criteria.

Much less suitable for explanation are methods associated with the computation of utility function gradients (e.g. Dyer-Gioffrion procedures, Dyer, 1976). This procedure necessitates a search for local gradients coefficients in different points. It is very difficult to characterize the logic of change in the direction of search and points of decisions along these directions in the decision space.

Admissible information processing operations

The majority of data processing operations, exercised by decision makers in man-machine procedures, can be classified in three groups: operations with names of criteria, operations with separate criteria estimates of one alternative, operations with alternatives presented as a set of criteria estimates. We shall refer to an operation as elementary if it cannot be partitioned to a larger number of operations relating to objects of the same group.

Elementary operations can be grouped in the following classes (Larichev et al., 1987; Larichev and Nikiforov, 1986):

(a) complex (C) if psychological research indicates that in performing these operations the decision maker is often inconsistent and/or makes use of simplifying strategies (e.g. eliminates a part of criteria);

(b) complex, except for small dimension problems (CS) if psychological research shows that the decision maker successfully performs these operations on small problems (2-3 criteria, 2-3 alternatives), but on larger problems he is often inconsistent and/or employs simplifying strategies;

(c) admissible (A) if the research indicates that the decision maker can manage them reliably , i.e. with a small number of inconsistencies, and using complex strategies (e.g. combination of several criteria estimates);

(d) uncertain (U, UC, UA) if an insufficient number of studies on these operations have been conducted but it is possible to judge about them by analogy (UC, UA).

The analysis of different man-machine procedures helped distinguish a small number (about 10) of elementary operations (see Overview, Larichev et al., 1987). A thorough examination of psychological literature made it possible to distinguish a cluster of procedures using only correct elementary operations of information elicitation from decision makers. All of them relate to a class of search for satisfactory criteria values (IMGP (Nijkamp and Spronk, 1980), STEM (Benayon et al., 1971), etc.).

Hence, man-machine procedures using a search for a pairwise tradeoff between criteria are more correct in terms of information elicitation from decision makers.

Human errors in search process

The use of correct operations of information elicitation from decision makers essentially reduces chances of errors or employment of simplified strategies by decision makers. It is, however, impossible to completely rule out human errors, for they can be brought about not only by cognitive constraints but also carelessness or fatigue. Also, errors can emerge at a time of learning when decision maker has not yet arrived at a compromise between criteria following examination of feasible values domain. Accordingly, it is necessary to secure a low sensitivity of man-machine procedures to decision maker and expert errors. A good means for reducing that of experts, estimating alternatives against many criteria (given a discrete variety of alternatives) is an interval assessment method first suggested by R. Steuer (Steuer and Schuler, 1978).

Procedures where a random error does not eliminate the feasible values domain from consideration are known to have a reduced sensitivity to decision maker errors. We used this criterion (Larichev, 1987) in comparing several man-machine procedures. Six out of 19 considered procedures did not meet this criterion.

It should be noted that all procedures give inadequate attention to possible decision maker errors. The methods of decision maker check for consistency used in a number of decision methods with subjective models (Gnedenko et al., 1979) boil down to duplication (directly or indirectly) of information elicited from decision maker.

Decision maker learning in the process of search

Everybody who has ever employed man-machine procedures for solving multicriteria mathematical programming problems knows that at the early steps of the procedure decision maker wants "everything at once", i.e. is willing to reach extremum against all criteria at a time. Only after familiarizing himself with the domain of feasible solutions he comes to understand the impossibility of this, and starts developing a more realistic approach.

All man-machine procedures, to some or other extent, provide opportunities for decision maker learning. Some of them, however, are better than others. Assuringly, at a time of learning decision maker should rather explore capacities of the extreme criteria values by reviewing criteria in turn rather than concurrently. This opportunity is provided by the methods of search for satisfactory criteria values.

Directions of further search

The procedures of MLPP solution, accounting for the specifics of different practical problems, have a long history. The recent overviews comparing man-machine procedures (Wallenius, 1975; Larichev and Polyakov, 1980; Polishchuk and Mirkin, 1980; Larichev and Nikiforov, 1986) reflect a desire to develop MLPP solution techniques with regard to the necessary criteria of their quality.

The existing man-machine procedures of MLPP solution get increasingly sophisticated. Still they are capable of advancing further. It is necessary to improve three basic components of man-machine procedures:

1. Methods of information elicitation from man with regard to specifics and limitations of human information processing system.

   The advances in solving this problem will make it possible to scientifically validate man-machine procedure in terms of psychological, mathematical, and informatics criteria.

2. Organization of an effective man-machine interface.

   There is a need for analysis of different types of information presentation. A special attention should be given to graphical images — cross-section of multicriteria space (Lotov, 1972), trajectory of "a Pareto race" (Korhonen and Wallenius, 1986).

3. Methods of effective solution of mathematical programming problems.

   The methods of multicriteria mathematical programming problem solution are based on iterative solution of single criterion problems. They can be rather complex: discrete, discrete-continuous, integer, etc. There is a need for methods of rapid solution of these problems with an acceptable accuracy (which is highly important for NP-complex problems). Otherwise we shall fail to maintain man-machine interaction.

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