8 A Practical Methodology of Solving Multicriterion Problems with Subjective Criteria

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8.1 INTRODUCTION

One typical problem in surveys of decision making methods is the small scale of current practical applications of these methods (Clarke, 1974; Larichev, 1974). There are many reasons for this. The path between the beginning of the practical work and the possibility of application of a method is long and difficult. This path is interesting by itself, but most of its stages depend on the specific circumstances. In the following sections, possible factors hindering successful application of a number of decision-making methods are discussed, and directions for development of promising new methods are suggested.

8.2 THE PROBLEM

The decision-making problems under discussion here have the following characteristics: (a) the decisions are nonrepetitive; (b) the criteria for evaluating the alternatives are subjective in that they can be defined only by the decision maker(s); and (c) the alternatives can be evaluated in terms of these criteria only by experts.

This class includes strategic R&D planning, or selection of directions, topics, or projects (Clarke, 1974; Larichev, 1974). The manager of an agency responsible for strategic R&D planning has a policy that is expressed, above all, in a list of criteria for evaluating lines of research or specific R&D projects. Most criteria are essentially qualitative – for example, "skill of presumed participants" or "project status." In estimating the alternative versions by these criteria, the decision maker should use the advice of experts. The situation in which the problem is solved is new each time, so no universal or standard solutions can be developed.

For many administrative and planning bodies, a large number of alternatives to evaluate and select among is increasingly common. This makes it hard for a manager who wishes to retain his control over decision making even if he cannot evaluate the proposed alternatives himself.

8.3 THE MEASUREMENTS

For each criterion, a list of evaluations should be made, resulting in an evaluation scale. The criteria formulated by a decision maker represent his attitude toward the problem of selection. Because the decision maker has to use the advice of experts in evaluating the alternatives, these evaluative criteria and scales are the language of communication between the decision maker and his experts. Even with the best possible experts, the result is largely dependent on the way the data are secured from them.

In my view, quantitative scales of evaluation are quite inapplicable to subjective criteria. With subjective criteria a 10-point scale does not enable a decision maker to obtain any reliable data, since each expert will have his own idea on the grade of quality to be assigned to each point.

The only reasonable approach is to use discrete scales of evaluations with a small number of qualitative evaluations in the form of verbal formulations of quality grades. These formulations also represent the decision maker's policy and his desire to distinguish certain qualitative differences in terms of a specific criterion. The formulations should be sufficiently detailed that experts can understand which grades are important to the decision maker. Evaluations on such scales, including the definitions of the best and the worst, represent the opinion of the decision maker; with another decision maker, these estimates might be quite different.

Table 8.1, adapted from Filippov *et al.* (1974), shows a possible scale for a very involved criterion, "promise of the line of research." A detailed formulation is needed to make the content of each estimate understandable to a set of experts.

Such scales greatly increase the confidence of decision makers in the data provided by the experts. Even though he agrees with quantitative scales, a rationally minded decision maker still does not completely rely on them because he understands the complexity, novelty, and ambiguity of measuring qualitative, subjective notions in numerical terms. Verbal scales enable the decision maker to request from the experts exactly what he needs.

8.4 SUBJECTIVE SELECTION RULES

The problem of evaluating the multicriterion alternatives will be solved when relations of utility are determined for all or some combinations of estimates in terms of the criteria. With subjective criteria these relations can be obtained only on the basis of knowledge of the decision maker's goals and preferences. Indeed, the degree to which evaluations in terms of partial criteria are combined in the

TABLE 8.1 Estimating the Promise of Basic Scientific Research in a Field (Probability of Breakthroughs)[a]

Criterion	Definition
B_1	World scientific consensus is that breakthroughs are highly probable and can lead to new theories and experimental methods.
B_2	World scientific consensus is that there are grounds (enough "mature," well-posed theoretical and experimental problems) to allow the formulation of more general theories, qualitatively different approaches to the description of the object of studies, new principles of experimental studies, and new scientific schools.
B_3	World scientific consensus is that steady growth, accumulation, and generalization of theoretical results and improvement of principles and methods of experimentation will continue over the next 5 to 10 years.
B_4	World scientific consensus is that there is little probability of qualitative changes; there is little innovation in approaches and research methods, and none is likely.
B_5	World scientific consensus is that further basic research in this direction will lead nowhere; the probability of reorienting the theories and research methods in this direction is very low.

[a] Adapted from Filippov *et al.* (1974).

overall evaluation cannot be determined through impartial computation. It is in comparison in terms of combinations of evaluations that the decision maker's goals and his attitude toward the problem of selection are represented.

The relations of different combinations of evaluations leading to the desired form of presenting the final decision will be referred to as the subjective model of decision making.

How subjective decision-making models for many criteria should be developed is a difficult question. There is a classical way (Morris, 1968) to determine preferences by comparing the utility of different lotteries; this approach, however, has been the object of valid criticism (Hall, 1965). It has been shown that in real situations people do not act in compliance with the preferences revealed by this technique (Dolbear and Lave, 1967). Apparently, even if people take the experiments seriously, they are aware of the wide difference between actual and model situations. Also, people tend to make mistakes in determining subjective probabilities (Tversky and Kahneman, 1973). In my view, the classical way of revealing preferences is especially ill-suited to unique, nonrepetitive decision-making problems.

The above technique is not the only one in which decision maker's preferences are used; there are also man–machine methods of decision making (Larichev, 1971), but these methods disregard descriptive data on the possibility of obtaining reliable information for people. In most cases, man is implicitly presumed to be omnipotent. Clearly, any preference-revealing procedures should rely on psychological and psychometric data on possibilities of elicitation of reliable information from people in different situations.

The hypothesis that people respond probabilistically in solving comparison or selection problems seems to be well tested (Luce *et al.*, 1963): there is a high probability that consistent answers will be obtained for a certain kind of problem, but these answers are not necessarily correct.

Many authors (Marschak, 1968; Mirkin, 1974) agree that the basic reason for the observed violations of the transitivity of preferences is the large number of attributes in the objects to be compared, which cannot be tackled by a single person at one time. This hypothesis also seems to be well tested.

These two hypotheses lead to the following suggestions for procedures for finding the preferences of decision makers and for procedures for design of subjective decision-making models. Above all, these procedures should include stability and consistency tests of decision maker's preferences. Also, the procedures should use those questions for which the probability of obtaining reliable information is the highest. Hypotheses concerning the possibility of obtaining the desired information from a decision maker should be formulated and tested.

Let us give one example. Scales of different criteria are taken in pairs [with N criteria, the number of such pairs is $\frac{1}{2}N(N-1)$]. Let the evaluations according to all the criteria be ordered, with performance decreasing from the beginning to the end of the scale. The hypothesis is as follows: with $N \leqslant 6$ or 7, the decision maker can, with small violations of transitivity, compare the performance deterioration in the scales of two criteria with the best estimates in terms of the other criteria.

To test the hypothesis, suppose we have the following scales of the criteria A and C:

Criterion A: R&D Project Status

A_1 The considerable amount of work necessary for the project has already been completed. No essential difficulties are expected in the remaining work.

A_2 A number of essential difficulties have to be overcome to complete the R&D project, but ideas on how to solve them exist and directions of research have been specified.

A_3 A number of novel little-known problems have to be solved, and no ideas or proposals on their solution are available.

Criterion C: Necessary Resources

C_1 No additional resources are required for the project; only some organizational arrangements are needed.

C_2 More personnel and material resources are needed within the framework of existing laboratories.

C_3 New laboratories should be set up for the project.

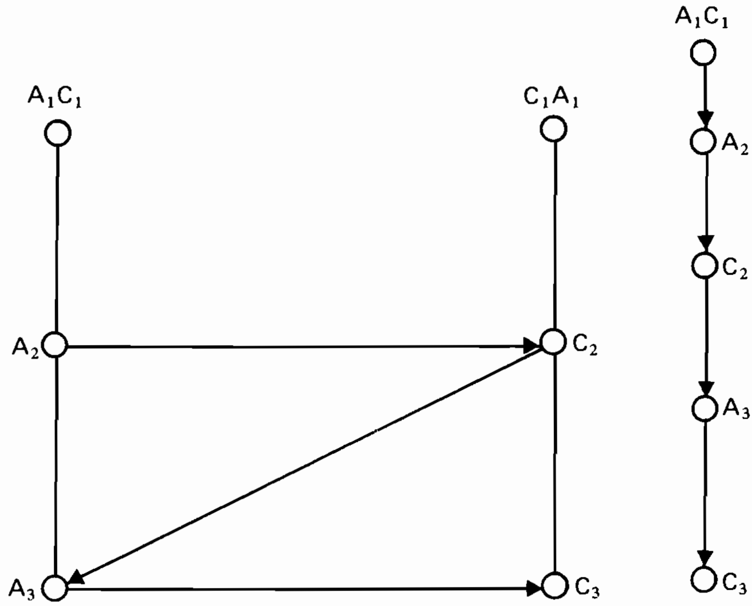

FIGURE 8.1 Uniform scale of criteria A and C.

The decision maker is told that initially the object has the highest values in terms of all the N criteria. Let us take up two cases: (a) the performance in terms of the criterion A has deteriorated (the value A_2 should be used instead of A_1); and (b) the performance in terms of the criterion C has deteriorated (the value C_2 should be used instead of C_1). The question is which of the cases is associated with a greater deterioration in performance. The answers were used to plot the graph of Figure 8.1 where the arrow denotes better performance; by this means, two scales may be replaced by one scale of estimates for the two criteria A and C. In this way, all pairs of criteria are considered. The information needed to obtain a single scale for the N criteria is duplicated, with the amount of redundant information increasing with the number of criteria. That redundant information can be used to test the above hypothesis.

The data obtained in testing the above hypothesis confirm its validity. Thus, no violation of transitivity was observed in questioning four decision makers with four criteria and from three to five qualitative estimates on the scales. In questioning with six criteria, two answers out of fifty-six were contradictory.

What is important is that all answers were concerned with real situations. The decision maker used his language in verbal evaluations for description of the situation, and he consistently pursued his goals in making the comparisons. The

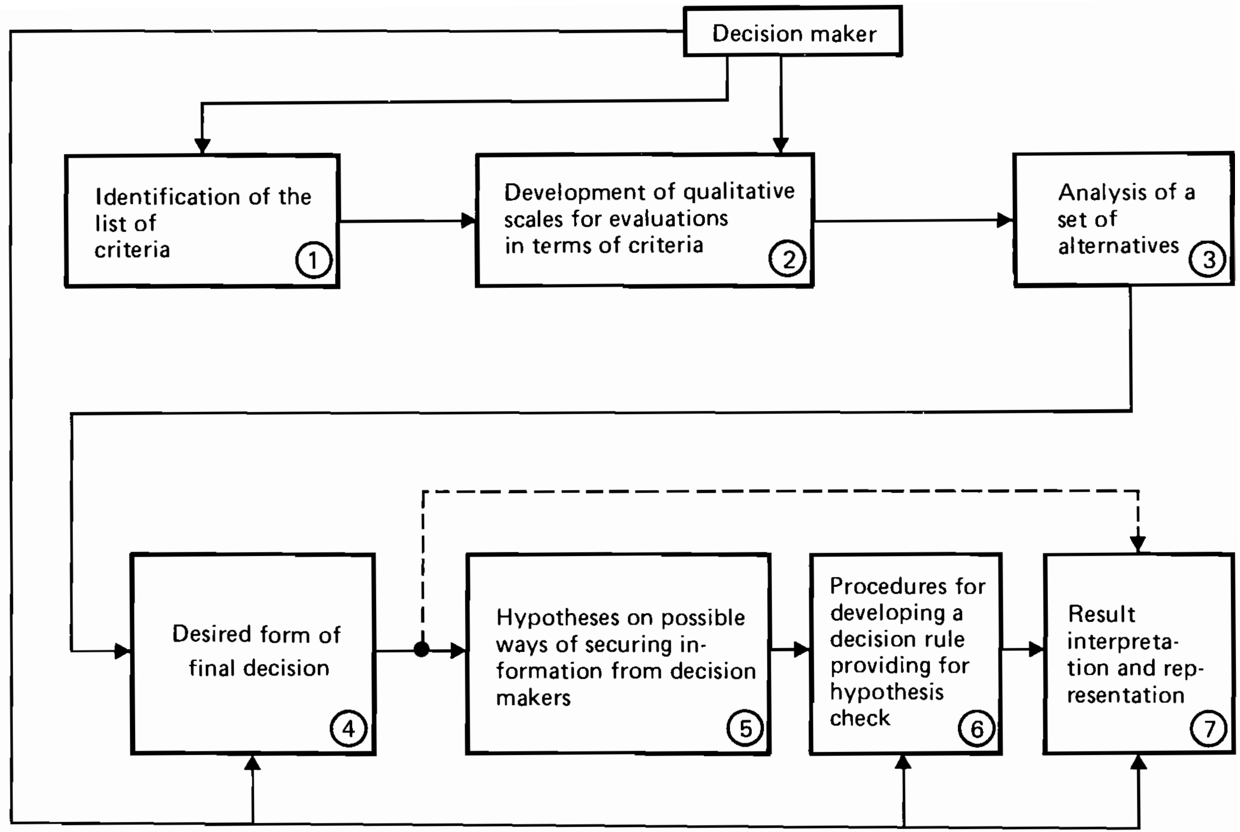

FIGURE 8.2 Stages of decision making with subjective criteria.

results of testing this hypothesis indicate that comparisons of performance deteriorations in pairs of scales can be used in procedures to reveal the preferences of decision makers (see, for example, Larichev *et al.*, 1974b).

8.5 STAGES OF DECISION MAKING WITH SUBJECTIVE CRITERIA

The need for separate stages for identifying the list of criteria and development of qualitative verbal scales of evaluations was discussed above (Figure 8.2). Special attention should be given to stage 3. If the set of multicriterion alternatives is specified (e.g., expert evaluations are obtained for the alternatives under study), then it would be useful to analyze that set by the methods of clustering (Ayvazian *et al.*, 1974) and decreasing the dimensionality of data (Teryokhina, 1973). In a number of cases this analysis may affect the initial requirements of decision makers as to the form in which the final decision should be represented, and in certain cases analysis may result in immediate solution of the overall problem. If a decision maker wants the set of alternatives to be divided into a small number of classes and analysis shows that they decompose into three groups, then it would be logical to assume that a final decision would require division of the alternatives into three classes. If analysis also reveals that the resulting three groups are in domination relation (each object in the first group is better than each one in the second group in terms of all criteria, and each object in the second group is better than each one in the third group in terms of all criteria), then the desired decision has been obtained.

The desired form of the final decision significantly affects the problem of decision making with subjective criteria. The most common forms are listed below:

- The alternatives are divided into two groups (with the better one identified).
- The alternatives are divided into a small number of groups.
- The alternatives are divided into a number of groups approaching, if possible, the number of combinations of criterion evaluations.

At stages 5 and 6 the data of preceding stages are used to develop the overall procedure for design of the decision rule, with the decision maker's preferences taken into consideration. Hypotheses are formulated on possible ways to obtain the data from decision makers; also formulated are ways to check these hypotheses. The chief difficulty is in constructing procedures that can lead to a representation of the final decision that incorporates those hypotheses that are the simplest and easiest to check.

8.6 AN EXAMPLE

The proposed approach was used in developing a method for planning applied research and development (Larichev *et al.*, 1974a,b), a method for selecting promising directions of basic research (Filippov *et al.*, 1974), and a method for estimating the quality of scientific manuscripts (Larichev and Glotov, 1974).

One of these methods was applied to a problem (Larichev *et al.*, 1974a) that can be regarded as portfolio optimization where the criterion of maximal economic efficiency can be applied. In reality, however, a planning body recognizes a number of qualitative criteria as well as cost and economic efficiency. Following the identification of a list of criteria and development of scales, all R&D projects were divided into two groups:

Those for which the effect of qualitative criteria dictates incorporation in the plan in some version – especially important projects (EIP).

Those whose inclusion in or exclusion from the plan depends on the indices of costs and cost effectiveness – common projects (CP).

The alternatives should thus be divided into EIPs and CPs. Note that in the practical cases under consideration analysis of data by dimensionality-reducing methods has revealed a large number of groups. To develop a subjective decision rule for dividing the objects into two classes, the following hypothesis was put forward:

Hypothesis 1 With $N \leqslant 6$ and two classes of final decisions (CPs and EIPs) the decision maker can in a stable way (with good repeatability with repeated questioning) and consistently (with rare violations of transitiveness) assign classes of final decisions to all combinations of estimates of two criteria under the assumption of best estimates in terms of the other $N-2$ criteria.

As an example consider combining criteria D and F defined as follows:

Criterion D: Social effect of implementing an R&D project

D_1 The project will have a direct and very large effect on improving the living standard of the population.

D_2 The project will make a direct contribution to improving the living standard of the population.

D_3 The project will make no direct impact on the living standard of the population.

Criterion F: Expected results compared with world standards

F_1 Expected results will surpass world standards.

F_2 Expected results will be on a par with world standards.

F_3 Expected results will be below world standards.

The information required from the decision maker is given in Figure 8.3. To check Hypothesis 1, one can use data redundancy resulting from treating all pairs of criteria and repeated questioning of decision makers over a period long enough to

	F_1	F_2	F_3
D_1	EIP	EIP	EIP
D_2	EIP	CP	CP
D_3	CP	CP	CP

FIGURE 8.3 Information required from decision maker. See text for definitions.

allow them to forget their earlier estimates when there are many of these (1 or 2 weeks).

In developing a decision rule, the following generalization may be used: with deterioration in estimations the performance of R&D projects does not increase. In other words, if some combination of estimations is associated with CPs, then all combinations dominated by that combination also belong to that class. When the estimations are binary and combinations of worse values of any three criteria are necessarily associated with CPs, the data from decision makers who fill in tables similar to Figure 8.3 for all pairs of criteria are sufficient for developing a decision rule. One can arrive at this particular case by combining the estimates of criteria that have the same effect on the quality of projects.

In a general case the information obtained through Hypothesis 1 is insufficient for obtaining a decision rule. There may be combinations of estimates with indefinite classes of quality. In this case, the following step can be proposed: boundaries that divide projects into EIPs in the space of N criteria should be checked in compliance with the following hypothesis.

Hypothesis 2 With $N \leqslant 6$ and two or three classes of final decisions (in our case two classes, EIPs and CPs), the decision maker can consistently compare projects differing in evaluations in terms of two criteria.

R&D projects that are on the boundary between EIPs and CPs are compared with other, nondominated ones whose evaluation differs in terms of two criteria. Let the character $\rightarrow$ denote better quality. Evidently, for the projects P_1 and P_2, if $P_1 \rightarrow P_2$ and $P_2 \in$ EIP, then $P_1 \in$ EIP. If $P_1 \rightarrow P_2$ and $P_1 \in$ CP, then $P_2 \in$ CP. Hypothesis 2 is checked through numerous comparisons to obtain redundant information.

In actual design of the decision rule by the above method (with $N = 4, 5,$ or 6) the number of combinations associated with EIPs was not very large. Therefore, only the data obtained through Hypothesis 1 could be used. That information was consistent, which confirms Hypothesis 1.

8.7 CONCLUSIONS

One characteristic feature of many weakly structured problems (Opther, 1965) tackled by systems analysis is the subjective nature of their models. Neglect of this fact and desire to obtain "pseudo-objective" models is one of the chief causes of the failure of practical application of systems analysis methods and theories to a wide range of problems (Schlesinger, 1963).

There are many problems in which qualitative, little-known, and uncertain aspects tend to dominate. Solution of these problems may be made easier if a method suitable to a particular decision maker or a group of decision makers is devised and if a special-purpose language is developed to enable the decision maker to express his policy and preferences as a subjective decision-making model.

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DISCUSSION

KEENEY: What experiences have you had in trying to generate subjective scales for real problems? Could you give me an example of one case where you have done that?

LARICHEV: I can refer to Figure 8.1. The two criteria, the amount of work necessary to complete a project and the resources necessary for this completion, were part of an analysis with real decision makers.

EDWARDS: I may have misunderstood, but I think you said that decision makers prefer to express their opinions in a rather small number of categories and, indeed, that they can do so only in this way, and that it is therefore undesirable to present them with a larger response set. Did I misunderstand that?

LARICHEV: The number of evaluation points on each scale depends on the particular case. We are trying – with the help of a decision maker – to find what the desirable number of evaluation points is in real situations. Sometimes you cannot place more than 5 points on a qualitative scale. In our practice we have usually had no more than 6. Of course, it is much easier to expand this if we have a quantitative scale.

RAIFFA: Suppose you have the situation depicted below, in which you write down your alternatives x and y and have criteria A, B, C, and D where levels of each of these are characterized by a verbal description.

	Criteria			
Alternatives	A	B	C	D
x	A_2	B_1	C_4	D_6
y	A_1	B_3	C_2	D_4
w	A_2	B_1	C_2	D_4

You could ask decision makers directly how they would feel about x and y. As another approach, do you ever create a hypothetical choice w, where w might be (A_2, B_1, C_2, D_4)? Notice that w is designed so that it makes it easy to make a choice between y and w because they are equivalent on criteria C and D. Similarly, x and w are equivalent on criteria A and B. It may be clear to the decision makers that y is better than w, and it may be clear that w is better than x. Therefore, creating a hypothetical situation might help the decision makers conclude that x is better than y.

LARICHEV: Of course in this case you can utilize this approach, but, as you said, it is easy now because you have the difference in only two criteria. In our problems, when there were many alternatives, it was impossible for decision makers to compare all the alternatives directly. It was necessary for us to utilize ways that directly compare only some of the alternatives that differ in a few criteria. Each problem requires a new decision on the best manner to do this. I think this is a very important question.

RAIFFA: When you compare pairs on criteria A and B, are you holding C fixed?

LARICHEV: I am holding C fixed at the best evaluation, which I describe verbally to the decision maker.

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